Beta Uncertainty and the Cross Section of Stock Returns

Dennis J. Lasser[1] and Andrew Lynch[2]
Binghamton University

Abstract

This paper examines to what extent the significance of size as a factor loading in asset pricing models is a proxy for the level of uncertainty in estimating an assets beta. We find that the time series variation of a stock's beta is highly correlated with firm size and appears to subsume size as a significant variable in the cross-sectional variation of asset prices. We show that the standard deviation and kurtosis of yearly beta estimates are: 1) negatively correlated to firm size, 2) outperform size when used as explanatory variables in cross-sectional regression estimates of both individual and portfolio asset return and 3) render size as insignificant when all are included in individual asset estimates and 4) have larger explanatory coefficients relative to size in the portfolio estimates. We therefore conclude that the uncertainty of beta is a significant reason for size representing a systematic factor in the Fama-French cross-sectional framework.

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[1] 326 Academic Building A, Binghamton University, 4400 Vestal Parkway E, Binghamton, NY 13902, dlasser@binghamton.edu, 607-777-4874
[2] 224 Academic Building A, Binghamton University, 4400 Vestal Parkway E, Binghamton, NY 13902, lynch@binghamton.edu, 607-777-3170

I. Introduction

A fundamental concern of the implementation of the CAPM as introduced by Sharp (1964) and similar asset pricing models is the stability of the factor beta estimations. Extant research with regard to CAPM beta estimates starting with Blume [3](1973) show that both individual and portfolio betas vary over time. Work by Chen (1981) and Chen and Keown (1981) among other show that the non-staionarity of the beta estimates may lead to misspecification in the CAPM framework. Ferson, Kandel and Stambaugh (1987) test conditional asset-pricing models that allow for variation in risk premiums and market betas. While their results reject to concept of conditional mean variance efficiency, they find the results to be strongest in a sub-period from 1963 through 1967where the size-effect appears to be most prominent. Alternatively, Bollerslev, Engle and Woolridge (1987) and Harvey (1989) among others show that conditional covariances do change over time and that asset pricing patterns may not be captured by the CAPM.

In this paper we examine whether uncertainty concerning time series variation in betas impacts asset prices. There are two potential sources of beta uncertainty, both of which will have an impact on expected returns. The first is the time varying nature of beta. A sizeable literature demonstrates that because market risk premia change over time (Shiller (1984), Lettau and Ludvigson (2001)) stock betas change over time (see e.g. Bollerslev, Engle, and Wooldridge (1988), Lettau and Ludvigson (2001)). A change in beta will result in a change in the stock's price. This potential systematic change in price is therefore a risk born by the investor that cannot be diversified away, creating higher expected returns in stocks with higher beta

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[3]Many studies have identified the instability of beta for examples papers by Altman, Jacquillat, and Levasseur, Baesel ,Fabozzi and Francis, Levitz, Levy , Roenfelt, Griepentrog, and Pflaum, .

uncertainty.[4] The second source of beta uncertainty stems from estimation error. As discussed by Roll (1986) it is impossible to verify the accuracy of CAPM due to the use of market return proxies. To the extent the market return proxy is incorrect, the resulting beta estimates will be less accurate. The result of this reduced accuracy will be a wider distribution in beta estimates over time. Market participants relying on beta as a measure of systematic risk will require incentive to hold stocks whose betas are less accurate, resulting in higher expected returns.

We find evidence consistent with beta uncertainty impacting the cross section of expected stock returns. Uncertainty is generally considered not knowing the probability distribution of future outcomes (Miller (1977)). Given that the lack of a probability distribution is difficult to measure we proxy beta uncertainty as the second and fourth moment of the prior year's monthly beta time series. We posit that as the standard deviation and kurtosis of a stock's beta increase, there is increased uncertainty pertaining to future changes in beta. Sorting stocks into equal weighted decile portfolios annually, we find that stocks with high beta standard deviations outperform those with low beta standard deviations by as much as 2.33 percent per month. Multivariate cross-sectional regressions reveal a statistically and economically significant positive relationship between both beta standard deviation and kurtosis with future excess returns. A 10 percent increase in beta standard deviation results in an increase in monthly returns of 16 basis points and a one unit increase in kurtosis is associated with a 5 basis point increase in monthly returns.

As argued by Berk (1995), the relation of stock market capitalization to stock returns may simply be the result of a misspecified asset pricing model. The natural question that stems from our beta uncertainty results is whether the effect we have identified subsumes size in explaining

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[4] This argument is similar in structure to the theoretical argument for liquidity risk being priced (Pastor and Stambaugh (2003), Acharya and Pedersen (2005)).

the cross-section of stock returns. The size anomaly has been has been a growing fixture in the asset pricing literature ever since it was first documented by Banz (1981). The impact of size was elevated in importance in the asset pricing literature by Fama and French (1992, 1993, 1996). They found that size along with the ratio of book value to market value were more powerful predictors of asset returns then the covariance of an assets return to the market as described in the CAPM. They argue that the book-to-market effect is a proxy for relative distress level of each firm perceived by the market. Therefore, higher book-to- market ratios simply suggests riskier firms. Alternatively, Fama and French do not pose an economic rationale for why size appears to be a systematic risk factor. They simply motivate its use from previous research that found size to be an anomaly in the CAPM framework.

We find evidence that stock market capitalization (size) is related to the level of uncertainty in the firm's beta, and therefore, is a contributing factor to its significance as an explanatory factor in asset pricing. Firstly, we find a significant correlation between size and the second and fourth moments of the estimated distribution of historical betas. Secondly, using cross-section models as described in Fama-French (1992), we find the standard deviation and kurtosis of firm and portfolio betas to be highly significant explanatory variables. We also find that the size variable becomes insignificant when included in the individual firm estimates and has lower but still significant coefficient in the portfolio beta estimated.

The remainder of the paper is presented as follows. Section 2 describes our data and methodology sections 3and 4 respectively discuss our results and conclusions.

2. Data and Methods

For this study we utilize all common equity (sharecodes 10 and 11) reported in both CRSP and COMPUSTAT from 1962 through 2011. We impose a minimum share price of 1 dollar and require at least 11 monthly observations in CRSP each year to be included in our sample. Market capitalization and book value are measured as of the end of the prior year, so only stocks with prior year returns, shares outstanding, and balance sheet data are included. Our total simple has 2,596,198 total firm month observations, from 21,935 separate firms, averaging 5,395 firms per month. Market risk factors and the risk free rate are obtained from Kenneth French's website (Fama and French (1993)).[5]

We estimate firm-level betas for all stocks in our sample using rolling regressions. Every month the prior 12 months of excess returns are regressed on the three Fama French excess return factors (Fama and French (1993)), as in equation (1):

$$ret_{i,t} - rf_t = \alpha_i + \beta_{M,i}MKTRF_t + \beta_{SMB,i}SMB_t + \beta_{HML,i}HML_t + \varepsilon_{i,t} \tag{1}$$

where $ret_{i,t} - rf_t$ is stock i's return in excess of the risk free rate in month t, $MKTRF_t$ is the market's return in excess of the risk free rate in month t, SMB_t is the return on small capitalization stocks in excess of the return on large capitalization stocks in month t, and HML_t is the return on stocks with high book-to-market values in excess of stocks with low book-to-market value in month t. The result is a monthly time varying beta for each stock for each of the three factors.

Alternatively, we estimate betas for the three factors using portfolios sorted on firm characteristics. Equal-weighted portfolios on market capitalization and book-to-market are formed at the end of every June. Every June stocks are sorted independently into size and book-to-market quintiles and then matched to form 25 total portfolios. These portfolios are held until

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[5] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

the following July. Rolling regressions are used in the same manner as in Section 2.1., where portfolio excess returns are regressed on the three Fama French factors using the prior 12 month's returns. The factors loadings are then assigned to each individual stock based on its portfolio membership each month (as in Fama and French (1992)).[6]

The focus of this study is on the uncertainty of market betas. We use the distribution of past beta estimates as a proxy for the uncertainty pertaining to future beta changes. Therefore, for every month we calculate the first through fourth moments of market beta for each stock using the prior 12 months of betas. This implicitly creates a 24 month overlap in standard deviation, skewness, and kurtosis. We believe this method produces the most conservative measure of variability, as it biases our estimates of beta variability downward, as the overlap reduces time series variation.

Table 1 reports the time series means of the cross sectional summary statistics of firm-level realized factor loadings (betas), as well as the time series moments of market betas, and the size and book-to-market characteristics of the firms. As expected, the average market beta is close to 1 (0.96) with the majority of values falling between 0.10 and 1.76 (the 25th and 75th percentiles). However, there remains significant cross sectional dispersion in betas (in the extremes, under -3 and above 5). Additionally, we find considerable time series variation in beta. The mean standard deviation of market beta is 0.87, economically significant given the average market beta of 0.96. The distribution of beta appears, on average, not to be skewed either positively or negatively. Though the mean skewness at the 25th and 75th percentiles is -0.55 and 0.55 respectively, showing some stocks have non-normally distributed betas. Finally, we find low average kurtosis, suggesting small tails in beta variability over time.

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[6] We have repeated our analyses using alternate portfolio sorting methods, including pre-rank beta (prior 12 months) and the standard deviation of beta (prior 12 months). Using various combinations of the four measure to sort into either 25 or 100 portfolios produces qualitatively similar results.

[Table 1 about here]

Overall, we find substantial evidence which shows that firm-level betas vary substantially over short periods of time (less than 1 year). While this variation is large, the average distribution of the variation is symmetric and narrow (small tails).

3. Results

3.1 Correlations Estimates

Table 2 examines the correlations between firm-level betas (Section 2.1), the moments of market beta, and the characteristics of the firm. We find several interesting results. First, and foremost, is that market capitalization is inversely related to both the standard deviation ($\rho = -0.119$) and kurtosis ($\rho = -0.006$) of beta. This suggests that smaller stocks have betas which are more variable and have wider distributions. Given the prominence of the size anomaly in asset pricing literature, and the lack of a strong positive theoretical explanation for it (Berk (2005)), this is the first evidence we see that beta variability may be related to stock prices. Additionally, we find a strong relation between market beta and both standard deviation and skewness of beta. Higher betas are associated with a more variable and positively skewed beta. This suggests a possible interaction effect

between beta and beta variability in our examination of returns.

[Table 2 about here]

3.2 Cross-section results

We begin our cross sectional analysis by examining the returns to portfolios formed on standard deviation of beta. Every month we sort all stocks into decile portfolios and hold those

portfolios for the following month. Table 3 reports the raw and risk adjusted equal-weighted

decile returns. The first and second columns use portfolios sorted on firm-level betas, while the

third and fourth columns are formed using betas estimated from portfolios (as described in

Section 2). We find a strong, positive relationship between the standard deviation of beta and the

following month's returns. Looking at firm-level betas, we find that the lowest Bstd portfolio has

a raw (risk-adjusted) return of 1.02 (0.50) percent while the highest Bstd portfolio has a raw

(risk-adjusted) return of 3.36 (2.67) percent. This different, of 2.33 and 2.18 percent respectively,

is not only highly statistically significant, but economically significant as well. Additionally, the

portfolios show a monotonic increase in return as Bstd increases. Turning to portfolio-level

betas, we find a similarly strong relation. Though economically smaller, and not uniformly

monotonic, the top Bstd decile out performs the bottom Bstd decile by a raw (risk-adjusted) 1.44

(1.35) percent.

[Table 3 about here]

As previously identified, there is a significant relation between the variability of beta and

firm characteristics, most notably market capitalization. Additionally, given the relation between

beta and beta variability, it may be important to account for the interaction between both effects.

We therefore move on to multivariate cross-sectional regressions. Every month we regress stock

excess returns on the prior month's beta, beta variability, and firm-level characteristics. We then

take the time series means of the coefficients and calculate t-statistics from the time series. Table

4 reports the results.

[Table 4 about here]

Panel A of Table 4 reports results using stock-level estimated betas. In Model (1) we find

expected base-line results. Market beta, estimated in a three factor model, is insignificant, while

book-to-market is positively related to future returns and market capitalization is inversely related to future returns. Substituting in the three moments of beta for size and book-to-market in Model (2) we find Bstd and Bkurt to be significantly positively related to the following month's returns. More importantly, however, we find in Model (3) that when we add market capitalization and book-to-market back into the model Bstd and Bkurt absorb the statistical and economic significant of market capitalization. This finding is important for two reasons. First, our portfolio sorting analysis in Table 3 reveals with returns adjusted for size that the Bstd effect is, at a minimum, in addition to the size effect. However, in finding that size becomes insignificant when the moments of beta are included in the model suggests that the size effect is merely a subset of the effect beta variability has on expected returns.

We find an additional interesting result in Model (6) when we include the interaction between beta and the moments of beta. The impact of Bstd on future returns is mitigated, somewhat, as beta increases. This result is partially intuitive. While beta does not have a technical upper boundary, over time high betas are more likely to reverse than increase. Interacted with a high standard deviation, such reversals are likely to be extreme, resulting in price increases as betas decline. Therefore, expected returns for high beta / high beta standard deviation stocks would naturally be lower. Additionally, Bskew, which on its own does not have a significant relation to future returns, positively impacts future returns as beta increases. This result also logically flows. Greater skewness means greater the right tail probabilities. Therefore higher betas with higher skewness mean a greater probability of beta remaining high (or even increasing), and higher expected returns. Panel B of Table 4 conducts the same analysis as Panel A using portfolio-level betas with results except that Kurtosis now inexplicitly becomes negative.

4. Conclusions

In this paper we find that uncertainty concerning a stock's beta affects asset pricing. Both portfolio sorting and cross sectional multivariate regressions reveal a statistically and economically significant positive relation between the standard deviation and kurtosis of beta and subsequent stock returns. Additionally, we find that beta uncertainty explains size as a priced factor in the cross section of stock returns. We find that beta uncertainty is highly correlated with firm size and appears to be an explanation for size being a significant variable in the cross-sectional variation of asset prices. The results show that the standard deviation and kurtosis of yearly beta estimates are: 1) negatively correlated to firm size, 2) outperform size when used as explanatory variables in cross-sectional regression estimates of both individual and portfolio asset return and 3) render size as insignificant when all are included in individual asset estimates and 4) have larger explanatory coefficients relative to size in the portfolio estimates. We therefore conclude that the uncertainty of beta is a priced systematic risk and a primary reason for size representing a systematic measure or risk in the Fama-French cross-sectional framework.

Table 1: Beta Variability Summary Statistics

Time series means of cross sectional summary statistics of stock characteristics. Betas are estimated with rolling regressions using the prior 12 months of stock returns regressed on the Fama-French three factor model. Beta moments are calculated using the prior 12 months of stock betas. *Bstd* is the standard deviation of market beta, *Bskew* is the skewness of market beta, and *Bkurt* is the kurtosis of market beta. β_{SMB} and β_{HML} are the factor loadings on SMB and HML respectively. Size is market capitalization and is reported in millions of dollars. Size and Book-to-Market (B/M) are measured as of the end of the prior year.

	Mean	Stdev	P1	P25	P50	P75	P99	N
β_{MKTRF}	0.96	1.67	-3.30	0.10	0.88	1.76	5.69	5,410
β_{SMB}	0.92	2.49	-4.96	-0.41	0.70	2.05	8.34	5,410
β_{HML}	0.21	2.68	-6.97	-1.06	0.22	1.48	7.58	5,410
Bstd	0.87	0.69	0.15	0.44	0.69	1.08	3.43	5,409
Bskew	0.01	0.83	-1.95	-0.55	0.01	0.55	1.97	5,408
Bkurt	-0.02	1.63	-1.99	-1.14	-0.46	0.62	5.80	5,406
Size	1,213	5,788	3	32	121	520	20,767	5,410
B/M	0.85	2.05	-0.63	0.42	0.73	1.14	3.89	5,395

Table 2: Correlations
Time series means of cross sectional Pearson correlation coefficients of stock level characteristics. t-statistics are calculated from the time series of correlation coefficients and are reported below in italics.

	Beta	SMB	HML	Bstd	Bskew	Bkurt	Size
β_{SMB}	-0.169						
	-9.59						
β_{HML}	0.236	0.098					
	13.65	*5.97*					
Bstd	0.063	0.139	0.07				
	10.67	*25.41*	*1.38*				
Bskew	0.084	-0.010	0.020	0.006			
	10.38	*-1.82*	*3.28*	*2.65*			
Bkurt	-0.004	0.003	-0.002	-0.087	0.005		
	-1.60	*1.94*	*-0.91*	*-20.92*	*1.15*		
Size	0.007	-0.085	-0.020	-0.119	-0.000	-0.006	
	11.83	*-62.13*	*-20.10*	*-141.85*	*-0.29*	*-7.00*	
B/M	-0.036	0.012	0.053	-0.044	0.005	0.001	-0.053
	-21.46	*3.95*	*20.00*	*-15.63*	*3.57*	*0.78*	*-36.74*

Table 3: Portfolio Sorting
Stocks are sorted into decile portfolios every month by the prior month's Bstd. Stocks with a price below 1 dollar are excluded. Return is the time series mean return for each portfolio. Firm-level betas are estimated through rolling regressions on firm returns as described in Table 1. Portfolio-level betas are estimated through rolling regressions on beta and beta standard deviation decile portfolio returns (estimated from the prior 2 years returns). Alpha is the intercept from a regression of portfolio excess returns on the Fama-French three factor model. t-statistics for the difference between the top and bottom decile are reported in italics.

	Firm-Level Betas		Portfolio-Level Betas	
	Return	Alpha	Return	Alpha
1 (Low Bstd)	1.02%	0.50%	1.22%	0.65%
2	1.10%	0.55%	1.16%	0.59%
3	1.11%	0.54%	1.22%	0.64%
4	1.16%	0.57%	1.23%	0.63%
5	1.18%	0.58%	1.24%	0.62%
6	1.24%	0.64%	1.32%	0.69%
7	1.38%	0.75%	1.54%	0.92%
8	1.53%	0.86%	1.59%	0.96%
9	1.86%	1.19%	1.97%	1.36%
10 (High Bstd)	3.36%	2.67%	2.66%	2.00%
10-1	2.33%	2.18%	1.44%	1.35%
	6.21	*5.87*	*5.62*	*5.35*

Table 4: Cross Sectional Regressions
One month ahead stock excess returns are regressed on factor loadings and beta moments (as described in Table 1). Coefficients reported are the time series means of cross-sectional regression coefficients with t-statistics reported below in italics. Panel A reports regression results using betas estimated on a firm level. Panel B reports regression results using betas estimated by size/book-to-market decile portfolios formed annually every July as in Fama and French (1992). Note: Compare to Fama and French (1992) Table III.

Panel A	(1)	(2)	(3)	(4)	(5)
Beta	0.0004	-0.0009	-0.0005	0.0004	0.0010
	0.75	*-1.40*	*-0.79*	*0.37*	*0.89*
Ln(B/M)	0.0024		0.0035		0.0035
	3.45		*5.90*		*6.11*
Ln(Mktcap)	-0.0016		-0.0004		-0.0003
	-4.23		*-1.17*		*-1.04*
Bstd		0.0162	0.016	0.0196	0.0196
		6.34	*6.97*	*5.92*	*6.41*
Beta*Bstd				-0.0016	-0.0018
				-1.66	*-1.89*
Bskew		-0.0005	-0.0004	-0.0012	-0.0010
		-0.64	*-0.52*	*-1.41*	*-1.24*
Beta*Bskew				0.0007	0.0006
				2.33	*2.17*
Bkurt		0.0005	0.0006	0.0008	0.0008
		2.50	*2.78*	*2.59*	*2.86*
Beta*Bkurt				-0.0001	-0.0001
				-0.43	*-0.51*
Adj R^2	0.031	0.045	0.059	0.055	0.069

Panel B	(1)	(2)	(3)	(4)	(5)
Beta	0.0040	0.0033	0.0039	0.0017	0.0026
	275	*1.77*	*2.21*	*0.70*	*1.11*
Ln(B/M)	0.0028		0.0031		0.0031
	4.02		*5.12*		*5.13*
Ln(Mktcap)	-0.0014		-0.0010		-0.0010
	-3.77		*-2.91*		*-2.86*
Bstd		0.0211	0.0175	0.0122	0.0094
		4.68	*4.39*	*1.65*	*1.45*
Beta*Bstd				0.0062	0.0055
				1.38	*1.31*
Bskew		0.0006	0.0005	-0.0010	-0.0010
		1.73	*1.71*	*-2.50*	*-2.42*
Beta*Bskew				0.0014	0.0013
				4.47	*4.31*
Bkurt		-0.0006	-0.0005	-0.0003	-0.0003
		-4.66	*-4.40*	*-1.61*	*-1.51*
Beta*Bkurt				-0.0001	-0.0001
				-1.03	*-0.81*
Adj R^2	0.037	0.035	0.050	0.039	0.053

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