Speculation, Risk Aversion, and Risk Premiums in the Crude Oil Market*

Bingxin Li
West Virginia University

September 19, 2014

Abstract

Speculative activity in commodity markets has increased dramatically over the last decade. I investigate whether aggregate risk aversion and risk premiums in the crude oil market co-vary with the level of speculation. Using crude oil futures and option data, I estimate aggregate risk aversion in the crude oil market and find that it is significantly lower after 2002, when speculative activity started to increase. Risk premiums implied by the state-dependent risk aversion estimates are negatively correlated with speculative activity, and are on average lower and more volatile after 2002. These findings suggest that index-fund investors who demand commodity futures for the purpose of portfolio diversification are willing to accept lower compensation for their positions. Estimated state-dependent risk premiums have substantial predictive power for subsequent futures returns and outperform commonly used predictors.

JEL Classification: G13; G17

Keywords: Oil; Futures; Options; Speculation; Risk Aversion; Risk Premium.

*I am grateful to Kris Jacobs for his invaluable advice and suggestions. I also thank Peter Christoffersen, Praveen Kumar, Craig Pirrong, Thomas George, Stuart Turnbull, Rauli Susmel, Hitesh Doshi, Harry Turtle, Alexander Kurov, Naomi Boyd, and seminar participants at the University of Houston and West Virginia University for their helpful suggestions and comments.

1 Introduction

The commodity market has grown rapidly over the past decade and has become an increasingly important component of financial markets. For exchange-traded commodity derivatives, the Bank for International Settlements (BIS) estimates that the number of outstanding contracts increased from 13.3 million in December 2003 to 137.4 million in June 2013.[1] Crude oil futures and options are the most liquid commodity derivatives. In December 2013, WTI and Brent crude oil futures accounted for 48.4% of dollar value of the S&P GSCI commodity index. Understanding the risk preferences and trading activities of investors in the crude oil market is therefore of great interest. It allows us to infer relevant information about investors' expectations, and it is crucial for the purpose of pricing and risk management.

The commodity market has also witnessed structural changes over the last decade. Before the early 2000s, commodity markets were partly segmented from financial markets and from each other (Tang and Xiong, 2012). After 2002, financial institutions started considering commodities as a new asset class to strategically diversify their portfolios. Researchers ascribe this change to the crash in equity market, the negative correlation between commodity returns and stock returns documented in the literature (Greer, 2000; Gorton and Rouwenhorst, 2006; Erb and Harvey, 2006), and the emergence of new financial instruments, such as long-only commodity index funds (LOCF) (Irwin and Sanders, 2011). According to Tang and Xiong (2012), the total value of various commodity-related instruments purchased by institutional investors increased from an estimated \$15 billion in 2003 to at least \$200 billion in mid-2008.

Following these structural changes, the literature has started debating if the increase in commodity index investment impacts the level of futures prices and risk premiums. Irwin and Sanders (2010) argue that there is no direct empirical link between index fund trading and commodity futures prices, and that fundamental supply and demand have determined crude oil prices; Hong and Yogo (2012) and Singleton (2011) find that speculative trading activity causes price drifts and predicts futures returns; Hamilton and Wu (2011) document significant changes in risk premiums after 2005, when speculative activity dramatically increased in the crude oil market. To the best of our knowledge, the relationship between the speculation level in the crude oil market and the risk aversion of the market participants has not yet been studied.

This paper investigates the relationship between the level of speculative activity and the aggregate risk aversion (or the resulting risk premiums) in the crude oil market. To motivate my main hypothesis I analyze a stylized model with one commercial hedger and one financial speculator in the crude oil futures market. From the optimal futures positions of the hedger

[1] In comparison, the number of outstanding contracts for exchange-traded equity index futures increased from 59.0 million contracts in December 2003 to 96.8 million in June 2013.

and speculator, I find that the more risk averse a hedger is, the more short futures positions she would hold; while the more risk averse a speculator is, the less long futures positions she would hold. At equilibrium, the model suggests a negative relationship between the speculation level and aggregate risk aversion. As speculation increases, the aggregate level of risk aversion of market participants decreases, and risk premiums decrease accordingly.

The empirical investigation is motivated by this stylized model and focuses on testing the dependence of market risk aversion and risk premiums on speculative activity. Using WTI crude oil futures and option data from the Chicago Mercantile Exchange and traders' position data from the CFTC, I estimate the market risk aversion based on a density forecast ability test. Following Bliss and Panigirtzoglou (2004), I first assume the risk aversion parameter is stationary over time and estimate the value of risk aversion by maximizing the forecast ability of subjective density functions which are implied from risk neutral densities and the assumed utility function. The risk premium is inferred from the normalized difference between risk neutral density functions and optimal risk-adjusted physical density functions. I then estimate the market risk aversions for high and low speculation periods respectively. I find that the aggregate relative risk aversion estimated for the high speculation period is lower than that estimated for low speculation period. Risk premiums for the high speculation period are on average lower than those for low speculation periods as well.

To further test the relationship of market risk aversion and risk premium with traders' speculative activities, I subsequently use the speculation index as a state variable and estimate state-dependent market risk aversion. I find evidence of a negative correlation between the state-dependent market risk aversion and the speculation level. The state-dependent market risk aversion is positive on average; however, after 2002, as speculation increases, risk aversion is more volatile, decreases over time, and occasionally negative. With state-dependent risk aversion, implied risk premiums are more volatile and on average lower after 2002, and also occasionally negative.

My findings on risk premiums are similar to those of Hamilton and Wu (2011), who use a very different modeling approach. My results are consistent with their interpretation that index-fund buyers who demand commodity futures for portfolio diversification are willing to accept much lower risk compensation, or would even pay a premium. Usually, commercial hedgers who hold futures positions need to hedge their price risks and would like to pay a premium to their counterparty. The financial traders who take the other side of the contract will receive this premium. However, as more and more financial institutions regard commodities as a new asset class and invest in the commodity market to diversify their portfolios, it is possible that they would want to pay a premium for their speculative positions.

To the best of my knowledge, this is the first paper to estimate risk aversion implied by the crude oil market. Pan (2011) investigates investor befliefs and state price densities in the crude oil market. He finds that investors assign higher state prices to negative returns when there are higher dispersion of beliefs and the increase in speculation reinforces this effect. Other related literature that estimates the representative agent's degree of risk aversion is mainly focused on the equity index market (Ait-Sahalia and Lo 2000; Jackwerth 2000; Ait-Sahalia, Wang, and Yared 2001; Rosenberg and Engle 2002). This paper estimates the state-dependent risk aversion coefficients in the crude oil market and documents the evolution of resulting risk premiums as the market structure changes.

This paper also contributes to the existing literature by inferring risk premiums in the crude oil market and studying their properties. Estimated state-dependent risk premiums are negatively correlated with the speculation level. I further test the ability of state-dependent risk premiums and other predictive variables, such as lagged futures returns, lagged volatility, and the speculation index to forecast subsequent futures returns. Risk premiums implied by state-dependent market risk aversion have significant explanatory power in predicting next period's futures returns, and their predictive power is higher than that of other predictors commonly used in the literature.

The rest of the paper proceeds as follows. In Section 2 I present a model with a hedger and a speculator in the crude oil futures market and develop the main hypothesis. Section 3 introduces the methodology used to estimate risk aversion and risk premiums. Section 4 discusses the data. Section 5 reports the main results and discusses the properties of the estimated state-dependent risk premiums. Section 6 checks the robustness of my findings using an alternative methodology and Section 7 concludes.

2 The Model

Assuming the existence of one commercial hedger and one financial speculator in the crude oil futures market, I study an equilibrium model and investigate its implications. This model builds on Duffie and Jackson (1990) who consider only one agent, the hedger, in the futures market.

2.1 The Hedger

The model includes a hedger (or commercial trader), who is directly exposed to the underlying crude oil commodity and uses futures to hedge the price risks.

Let $B = (B^1, \ldots, B^N)$ denote a Standard Brownian Motion in R^N which is a martingale with respect to the agent's filtered probability space. The spot price of crude oil is given by

$$\frac{dS_t}{S_t} = \mu_t dt + \sigma_t dB_t \tag{2.1}$$

where μ_t and σ_t are the mean and variance process of crude oil spot returns at time t, with μ_t is 1-dimentional and σ_t is $(1 \times N)$-dimensional.

Assume there are K futures contracts available for trade. The futures prices are given by a K-dimensional Ito process F_t

$$\frac{dF_t}{F_t} = m_t d_t + v_t dB_t \tag{2.2}$$

where m_t and v_t are the mean and volatility process for the futures contracts at time t, with m_t is K-dimentional and v_t is $(K \times N)$-dimensional.

The hedger's total wealth is the sum of the terminal value of a fixed portfolio of spot market assets and the terminal value of a margin account on a futures trading position. It is given by

$$dW_t^{\theta_h} = \pi_{h,t} dS_t + dX_t^{\theta_h} \tag{2.3}$$

where $\pi_{h,t}$ is the hedger's physical position in crude oil at time t and $X_t^{\theta_h}$ is the margin account with

$$X_t^{\theta_h} = \int_0^t e^{r(t-u)} \theta_{h,u} dF_u \tag{2.4}$$

where $\theta_{h,t} = (\theta_{h,t}^1, \ldots, \theta_{h,t}^K)$ is the futures position strategy of the hedger at time t. A positive number for the hedger's physical position ($\pi_{h,t} > 0$) means that crude oil is in net supply, while a negative number ($\pi_{h,t} < 0$) means crude oil is in net demand. Similarly, a positive number for the hedger's futures position ($\theta_{h,t} > 0$) represents long positions in futures and a negative one ($\theta_{h,t} < 0$) represents short positions.

The hedger's problem is

$$\max_{\theta_h} E[U(W_T^{\theta_h})] \tag{2.5}$$

Assume the hedger's utility function takes the exponential form and her relative risk aversion at time t is γ_t^h, we can solve for hedger's optimal futures position

$$\theta_{h,t} = -\frac{(v_t v_t')^{-1}}{F_t}[v_t \sigma_t' \pi_{h,t} S_t - m_t/\gamma_t^h] \tag{2.6}$$

The proof is provided in the Appendix. The hedger's optimal futures position is the same as that of the single agent model in Duffie and Jackson (1990).

2.2 The Speculator

Now consider a speculator in this market who trades with the hedger for financial profits. The speculator (or financial trader) is not directly engaged in trading the crude oil spot commodity and instead uses crude oil futures for the purpose of making financial profits. The speculator does not hold the spot commodity. Her margin account is

$$X_t^{\theta_s} = \int_0^t e^{r(t-u)} \theta_{s,u} dF_u \tag{2.7}$$

where $\theta_{s,t} = (\theta_{s,t}^1, \ldots, \theta_{s,t}^K)$ is the speculator's futures position strategy at time t.

The speculator's total wealth is $dW_t^{\theta_s} = dX_t^{\theta_s}$, so her maximization problem is

$$\max_{\theta_s} E[U(W_T^{\theta_s})] \tag{2.8}$$

Similar to solving the hedger's problem, I assume exponential utilities for the speculator and denote her relative risk aversion as γ_t^s. Her optimal futures position is given by

$$\theta_{s,t} = -\frac{(v_t v_t')^{-1} m_t}{F_t \gamma_t^s} \tag{2.9}$$

2.3 Equilibrium

Market clearing requires $\theta_{h,t} + \theta_{s,t} = 0$. This implies

$$m_t = \frac{v_t \sigma_t' \pi_{h,t} S_t}{\frac{1}{\gamma_t^h} + \frac{1}{\gamma_t^s}} \tag{2.10}$$

Define the degree of aggregate absolute risk aversion of the representative agent, or the market risk aversion, as an average of the population degree of risk aversion. For simplicity, I assume they have equal weights[2]

$$\Gamma_t \equiv \gamma_t^h + \gamma_t^s \tag{2.11}$$

Substituting (2.10) and (2.11) back into (2.6) and (2.9), we get

$$\begin{aligned} \theta_{s,t} &= \frac{S_t (v_t v_t')^{-1} v_t \sigma_t' \pi_{h,t}}{F_t} \frac{\gamma_t^h}{\gamma_t^h + \gamma_t^s} \\ &= \frac{S_t (v_t v_t')^{-1} v_t \sigma_t' \pi_{h,t}}{F_t} \frac{\gamma_t^h}{\Gamma_t} \end{aligned} \tag{2.12}$$

[2] This can be easily generalized to a weighted average of the risk aversion of the hedger and the speculator. This generalization does not change our main conclusions.

$$\theta_{h,t} = -\frac{S_t(v_t v_t')^{-1} v_t \sigma_t' \pi_{h,t}}{F_t} \frac{\gamma_t^h}{\Gamma_t} \tag{2.13}$$

Equations (2.12) and (2.13) solve equilibrium optimal positions in futures contracts for the speculator and hedger respectively. Thus I obtain the following implications:

1. In equilibrium, traders' absolute positions in futures contracts are proportional to the covariance between the futures prices and spot prices, $v_t \sigma_t'$. A high covariance term indicates that futures contracts provide a good hedge, suggesting a high demand for hedging. When the prices of futures contracts and the spot commodity are perfectly correlated, a risk averse ($\gamma_t^h > 0$) producer ($\pi_{h,t} > 0$) would like to short futures contracts ($\theta_{h,t} < 0$) to hedge her price risk and a risk averse speculator ($\gamma_t^s > 0$) would take the other side of the contract ($\theta_{s,t} > 0$).

2. The absolute values of traders' positions in futures contracts ($|\theta_{h,t}|$ and $|\theta_{s,t}|$) are proportional to the hedger's net spot position $\pi_{h,t}$. The more physical crude oil is in net supply ($\pi_{h,t} > 0$ and $\pi_{h,t}$ increases), the more short futures positions the hedger would take for hedging purposes ($\theta_{h,t} < 0$ and $|\theta_{h,t}|$ increases), and the more long futures positions the speculator would take to offset the hedger's position ($\theta_{s,t} > 0$ and $\theta_{s,t}$ increases). On the other hand, the more physical crude oil in net demand ($\pi_{h,t} < 0$ and $|\pi_{h,t}|$ increase), the more long futures positions are required by hedgers for hedging purposes ($\theta_{h,t} > 0$ and $\theta_{h,t}$ increases), and the more short futures positions the speculator would take to offset the hedger's position ($\theta_{s,t} < 0$ and $|\theta_{s,t}|$ increases).

3. Traders' futures positions are negatively related to their level of risk aversion. The more risk averse a hedger is (γ_t^h increases), the more short futures positions she would like to hold ($\theta_{h,t} < 0$ and $|\theta_{h,t}|$ increases); while the more risk averse a speculator is (γ_t^s increases), the less long futures positions she would hold ($\theta_{s,t} > 0$ and $\theta_{s,t}$ decreases). Assume everything else equals, as the risk aversion of the speculator (γ_t^s) decreases, the market risk aversion (Γ_t) would decreases accordingly, and the speculative activities ($\theta_{s,t}$) would increase.

In the model, the level of market risk version is directly related to the market participants' trading positions. The equilibrium optimal position suggests a negative relationship between market risk aversion and speculative activity. As we observe more speculative activity in the crude oil market in recent years, we expect a lower aggregate market risk aversion.

Motivated by this model, I hypothesize that market risk aversion is state-dependent. When there is more speculation, market risk aversion is low. I test this hypothesis in the empirical analysis below.

3 Estimating Market Risk Aversion and Risk Premium

It would be interesting to estimate the risk aversion coefficients of the hedger and speculator separately, but this necessitates strong assumptions. Instead, I estimate the risk aversion coefficient of the representative agent, or the aggregate market risk aversion. I explain the methodology used to perform this estimation in 3.1 and 3.2. Two utility functions used in the empirical analysis are discussed in 3.3. In 3.4, I extend this methodology to allow estimation of state-dependent market risk aversion. I introduce the calculation of the resulting risk premium in 3.5.

3.1 Market Risk Aversion

According to asset pricing theory, the risk neutral density function is related to the objective density function by the representative investor's utility function. The representative agent's risk aversion is embedded in the utility function, given certain conditions such as complete and frictionless markets and a single asset (Ait-Sahalia and Lo, 2000; Jackwerth, 2000; Bliss and Panigirtzoglou, 2004; Christoffersen, Heston, and Jacobs, 2013). I estimate aggregate risk aversion by considering a representative agent in the crude oil market, and assuming that her wealth can be represented by the overall price level of the crude oil market.[3]

Assume that the representative agent's utility function is $U(.)$. At time t, the price of a crude oil futures contract maturing at time T, $F_{t,T}$, is given by

$$F_{t,T} = E\left[\beta \frac{U'(F_T)}{U'(F_{t,T})} F_T\right] \tag{3.1}$$

where $F_T = F_{T,T} \equiv S_T$ is the spot price at expiration, and β is the impatience factor.

Under the risk neutral measure,

$$F_{t,T} = E^Q\left[\exp(-r(T-t))F_T\right] \tag{3.2}$$

Defining the pricing kernel, $\xi(F_T) \equiv \frac{f^Q(F_T)}{f(F_T)}$, where $f(.)$ and $f^Q(.)$ are the physical and risk neutral density functions, we obtain

$$\xi(F_T) \equiv \frac{f^Q(F_T)}{f(F_T)} = \frac{\alpha U'(F_T)}{U'(F_{t,T})} \tag{3.3}$$

[3]One may wonder if the assumption of a representative agent in the crude oil market is valid since this market is generally regarded as incomplete. Duffie (2001) shows that if the gradient of an agent's utility function at an optimal-consumption process exists and is smooth-additive, the calculations for the representative agent can be repeated for each agent, and the market level of risk aversion can be calculated using equation (2.11). A model can have a representative agent when agents differ but act in such a way that the sum of their choices is mathematically equivalent to the decision of one individual or many identical individuals.

where $U'(.)$ is the marginal utility function and α is a constant.

From equation (3.3), given any two of the following three: the risk neutral density function, the physical density function, and the pricing kernel (or utility function), we can infer the third. For example, if we know the risk neutral probability density function, one can either assume a pricing kernel or a utility function to imply a physical density, or make an assumption on the physical density to infer the pricing kernel.

Estimating the representative agent's degree of risk aversion has a long history in the equity index market. The methodology in most studies is to separately estimate the risk neutral density from options prices and the objective (or statistical) density function from historical prices of the underlying asset. Use these two separately derived functions to infer the pricing kernel, and then draw conclusions for the implied utility function or risk aversion coefficient (Ait-Sahalia and Lo 2000; Jackwerth 2000; Ait-Sahalia, Wang, and Yared 2001; Rosenberg and Engle 2002).

Cross-sections of option prices have been widely used to estimate implied risk neutral probability density functions. These risk neutral probability density functions represent forward-looking forecasts of the distributions of prices of the underlying asset at a single point of time. The physical density function is more challenging to estimate. One cannot independently estimate a time varying statistical density from a time series of prices without imposing an a priori structure. For example, Jackwerth (2000) uses one month of daily return data and calculates 31-day, non-overlapping returns from sample. Ait-Sahalia and Lo (2000) use a relatively long series of overlapping returns to estimate the actual distribution. Christoffersen, Heston, and Jacobs (2013) obtain a conditional density by standardizing the monthly return series by the sample mean and the conditional one-month variance on that day.

Bliss and Panigirtzoglou (2004) point out that these studies impose assumptions of stationarity on the statistical density function or the parameters of the underlying stochastic prices which are not implied or required by the theory. They propose a different approach that assumes the risk aversion parameter is stationary over the sample period and estimates the value of risk aversion by maximizing the forecast ability of subjective PDFs, which are implied from risk neutral PDFs and the assumed utility function (or pricing kernel).

3.2 Estimating Constant Market Risk Aversion

In this paper, I estimate the market risk aversion parameter by adopting Bliss and Panigirtzoglou's (2004) PDF forecast ability method. If investors are rational, their subjective density forecasts should correspond, on average, to the distribution of realizations. The risk aversion coefficient in the utility function provides a measure of the degree of risk aversion of the representative investor in the crude oil market.

I now describe the estimation procedure for market risk aversion in more detail. Option prices embed risk neutral PDFs. Breeden and Litzenberger (1978) show that the risk neutral PDF for the value of the underlying asset at option expiry, $f(S_T)$, is related to the European call price by

$$f^Q(S_T) = e^{r(T-t)} \frac{\partial^2 C(S_t, K, t, T)}{\partial K^2}|_{K=S_T} \tag{3.4}$$

where S_t is the current value of the underlying asset, K is the option strike price, and $T - t$ is the time to expiry.

In the case of the crude oil derivatives data, I use the semi-parametric approach first introduced in Ait-Sahalia and Lo (1998) and follow the implementation of Christoffersen, Heston, Jacobs (2013) and Pan (2011). The risk neutral density for the spot price at the maturity date T is given by

$$\widehat{f}^Q(F_T|F_t) = e^{r(T-t)} \frac{\partial^2 \widehat{C}(F_{t,T}, K, t, T, \sigma(K,T))}{\partial K^2}|_{K=F_T} \tag{3.5}$$

where $\sigma(K,T)$ is the Black (1976) implied volatility.

Given an estimated risk neutral density function and a utility function, the implied subjective density function is

$$\widehat{f}(F_T) = \frac{\frac{\widehat{f}^Q(F_T)}{\xi(F_T)}}{\int \frac{\widehat{f}^Q(x)}{\xi(x)} dx} = \frac{\frac{U'(F_t)}{\lambda U'(F_T)} \widehat{f}^Q(F_T)}{\int \frac{U'(F_t)}{\lambda U'(x)} \widehat{f}^Q(x) dx} = \frac{\frac{\widehat{f}^Q(F_T)}{U'(F_T)}}{\int \frac{\widehat{f}^Q(x)}{U'(x)} dx} \tag{3.6}$$

I then use the Berkowitz (2001) probability density function forecast ability test to estimate the market risk aversion coefficient and infer physical probability density functions, as in Bliss and Panigirtzoglou (2004). Berkowitz (2001) proposes a parametric methodology for jointly testing uniformity and independence of the density functions. He defines a transformation, z_t, of the inverse probability transformation, y_t, using the inverse of the standard normal cumulative density function, $\Phi(.)$:

$$z_t = \Phi^{-1}(y_t) = \Phi^{-1} \int_{-\infty}^{X_t} \widehat{f}_t(s) ds \tag{3.7}$$

Under the null hypothesis, $\widehat{f}_t(.) = f_t(.)$, $z_t \sim i.i.d\ N(0,1)$. Berkowitz (2001) tests the independence and standard normality of the z_t by estimating the following equation using maximum likelihood:

$$z_t - \mu = \rho(z_{t-1} - \mu) + \varepsilon_t \tag{3.8}$$

I test restrictions on the estimated parameters using a likelihood ratio test. Under the null, the parameters of this model should be: $\mu = 0, \rho = 0$, and $Var(\varepsilon_t) = 1$. Denoting the log-likelihood function as $L(\mu, \sigma^2, \rho)$, the likelihood ratio statistic is

$$LR = -2[L(0,1,0) - L(\widehat{\mu}, \widehat{\sigma}^2, \widehat{\rho}) \tag{3.9}$$

which will be distributed $\chi^2(3)$ under the null hypothesis.

3.3 Two Utility Functions

First, I consider the power utility function

$$U(F_T) = \frac{F_T^{1-\Gamma} - 1}{1-\Gamma} \tag{3.10}$$

where Γ is the measure of market relative risk aversion (MRRA), $\Gamma = MRRA = -\frac{F_T U''(F_T)}{U'(F_T)}$. The corresponding pricing kernel is

$$\xi(F_T) = \frac{\alpha U'(F_T)}{U'(F_{t,T})} = \alpha \left(\frac{F_T}{F_{t,T}} \right)^{-\Gamma} \tag{3.11}$$

Substituting equation (3.11) into (3.6), I get

$$\widehat{f}(F_T) = \frac{\frac{\widehat{f}^Q(F_T)}{F_T^{-\Gamma}}}{\int \frac{\widehat{f}^Q(x)}{x^{-\Gamma}} dx} \tag{3.12}$$

I first choose an initial value of Γ, and then maximize the forecast ability of the resulting subjective probability density functions by maximizing the p-value of the Berkowitz LR statistic with respect to Γ. The forecast ability test of physical density functions gives out an estimate of the market relative risk aversion, Γ, and the corresponding risk-adjusted physical probability density functions.

Second, I use the exponential utility function throughout this paper for comparison. The exponential utility function is given by

$$U(F_T) = -\frac{e^{-\eta F_T}}{\eta} \tag{3.13}$$

where η is the market absolute risk aversion (MARA) with $\eta = MARA == -\frac{U''(F_T)}{U'(F_T)}$.The market relative risk aversion is ηF_T. The pricing kernel under the exponential utility is

$$\xi(F_T) = \frac{\alpha U'(F_T)}{U'(F_{t,T})} = \alpha e^{-\eta(F_T - F_{t,T})} \tag{3.14}$$

Substituting equation (3.14) into (3.6), I get the physical density function as

$$\widehat{f}(F_T) = \frac{\frac{\widehat{f}^Q(F_T)}{e^{-\eta F_T}}}{\int \frac{\widehat{f}^Q(x)}{e^{-\eta X}} dx} \tag{3.15}$$

3.4 Estimating State-Dependent Market Risk Aversion

The market risk aversion estimation approach introduced in Section 3.1 assumes the risk aversion parameter is stationary over the sample period and does not allow for state dependence of market risk aversion. However, Bliss and Panigirtzoglou (2004) document that the implied relative risk aversion in the equity market is volatility-dependent. Christoffersen, Heston, and Jacobs (2013) also develop a model with a variance-dependent price kernel and find a negative variance premium. As for the crude oil market, according to the analysis in Section 2, I am interested in investigating the possibility that financialization has changed the structure of the crude oil market. Therefore I make further assumptions and allow the market risk aversion coefficient to change over time.

To estimate the state-dependent market risk aversion, I assume that the market risk aversion is a linear function of a state variable x_t, which measures the level of speculative activity. For the case of the power utility function, I assume

$$\Gamma_t = a + b(x_t - \overline{x_t}) \tag{3.16}$$

where Γ_t is the state-dependent market relative risk aversion at time t. a is the average level of the overall market relative risk aversion. The slope coefficient, b, determines the variation of market relative risk aversion with the state variable x_t.

Similarly, in the case of exponential utility, I assume

$$\eta = a + b(x_t - \overline{x_t}) \tag{3.17}$$

where η is the state-dependent market absolute risk aversion at time t.

I estimate the values of coefficients a and b by running the Berkowitz (2001) density forecast ability test, maximizing LR p-values for the adjusted physical density functions, for both power and exponential utilities.

3.5 Implied Risk Premiums

The difference between the normalized means of the risk neutral probability density function and the subjective probability density function is an approximate measure of the risk premium (Bliss and Panigirtzoglou, 2004)

$$RP_t \approx \frac{E_t[\widehat{f}^Q(F_T)] - E_t[\widehat{f}(F_T)])}{E_t[\widehat{f}^Q(F_T)]} \tag{5.2}$$

Using the estimated risk aversion parameters and the risk neutral density function calculated from crude oil options we can obtain the implied physical probability density function. Then we can infer the resulting risk premium from the formula above. I calculate the risk premiums implied by a power utility function and an exponential utility function respectively. The results are reported in Section 5.

4 Data

4.1 Crude Oil Futures and Options Data

The Chicago Mercantile Exchange (CME group, formerly NYMEX) crude oil derivatives market is the world's largest and most liquid commodity derivatives market. The range of maturities covered by futures and options and the range of option strike prices are also greater than for other commodities (Trolle and Schwartz, 2009). I use a data set of WTI crude oil futures and options contracts traded on the CME from January 2nd, 1990 to December 3rd, 2008.

Futures contracts were screened based on patterns in trading activity. Open interest for futures contracts tends to peak approximately two weeks before expiration. Among futures and options with more than two weeks to expiration, the first six monthly contracts tend to be very liquid. For contracts with maturities over six months, trading activity is concentrated in the contracts expiring in March, June, September, and December. Due to these liquidity patterns, I filter the futures and options data as follows: I retain all futures contracts with six weeks or fewer to expiration; among the remaining, I retain the first five monthly contracts (M2-M6); beyond that, I choose the first two contracts with expiration either in March, June, September or December (Q1-Q2).

Figure 1 plots the prices of the filtered futures contracts with maturity from one month up to one year (M1-M6 and Q1-Q2). All prices in this paper are settlement prices.[4] To avoid cluttering of the figure, only the futures term structure on Wednesdays is displayed. From Figure 1, we can observe that futures prices have increased dramatically since 2003 and subsequently declined after July 2008. The prices of long maturity futures contracts, e.g., Q2 futures contracts, are lower on average than that of short maturity futures contracts, e.g., M1 futures contracts. Generally speaking, the crude oil market is in backwardation, consistent

[4]The CME light, sweet crude oil futures contract trades in units of 1000 barrels. Prices are quoted in US dollars per barrel.

with existing studies (Trolle and Schwartz, 2009; Litzenberger and Rabinowitz, 1995). It is worth noticing that in recent years, especially after 2005, the frequency of the crude oil market being in backwardation decreases gradually. Using one month (M1) futures contracts as a proxy for the spot prices, the one year futures contracts (Q2) are strongly backwardated 70.7% of the time before January 2005 and strongly backwardated 50.3% of the time after 2005. According to the normal backwardation theory in Keynes (1930), when the market is backwardated, we expect the risk premium to be positive; while when the market is in contango, we expect a negative risk premium.

To screen the options data, I first retain options on filtered futures contracts above. Because trading in options markets is asymmetrically concentrated in at-the-money and out-of-the-money options, and our methodology will not accommodate duplicate strikes in the data, I discard in-the-money options. Options for which they are impossible to compute an implied volatility (usually far-away-from-the-money options quoted at their intrinsic value), or options with implied volatilities of greater than 100 percent, are also discarded. If there are fewer than five remaining usable strikes in a given cross-section, the entire cross-section is discarded. Only those options that have open interest in excess of 100 contracts and options with prices larger than ten cents are considered. In addition, I exclude those observations with Black (1976) implied volatility less than 1% or greater than 100%.

Crude oil futures contracts expire on the third business day prior to the 25th calendar day (or the business day right before it if the 25th is not a business day) of the month that precedes the delivery month. Options written on futures expire three business days prior to the expiration date of futures. A target observation date is then determined for horizons of 1, 2, 3, 4, 5, and 6 weeks; 2, 3, 4, 5, 6, and 9 months; and 1 year by subtracting the appropriate number of days (weekly horizons) or months (monthly and 1 year horizon) from the expiration date, according to Bliss and Panigirtzoglou (2004). If there are no options traded on the target observation date, the nearest options trading date is determined. If this nearest trading date differs from the target observation date by no more than 3 days for weekly horizons or 4 days for monthly and 1-year horizons, that date is substituted for the original target date. If no sufficiently close trading date exists, that expiry is excluded from the sample for that horizon. Table 1 reports the summary statistics of the filtered options contracts for each forecast horizon.

We use American options on crude oil futures contracts. The CME has also introduced European-style crude oil options, however, its trading history is much shorter and liquidity is much lower than for the American options. Since the option pricing formula is designed for European options, we have to convert the American option prices to European option prices. I convert the American option prices to European option prices using the method in Trolle and Schwartz (2009). It consists of inverting the Barone-Adesi and Whaley (1987) formula for American option prices which yields a log-normal implied volatility, from which we can subsequently obtain the European option price using the Black (1976) formula.

Figure 2 plots the implied ATM volatility of options on futures contracts maturing in four weeks. The large spikes in the option implied volatilities appear around the end of 1990 and beginning of 1991 (which is the time of the first Gulf War), the September 2001 terrorist attack, the second Gulf War in March 2003, and during the financial crisis in 2008.

4.2 Trading Position Data

The Commodity Futures Trading Commission (CFTC) publishes trading positions of commercial hedgers and financial speculators twice every month before September 30, 1992 and once every week since then. Futures positions can be found in the futures only Commitments of Traders (COT) report. As defined in the report, hedgers are those investors who have direct exposure to the underlying crude oil commodities and use crude oil futures for hedging purposes, and speculators are those investors who are not directly engaged in the underlying crude oil commodities but use derivatives markets for the purpose of financial profits. This definition is in accordance with that proposed in the model of Section 2 and thus can be used as a measure of the trading position of crude oil market participants. Starting in 2006, the CFTC began to report positions of traders in a finer category: commercials, managed money, commodity dealers, and others. The fundamental distinction among traders as to whether they have physical attachment in the crude oil market or not still holds (Pan, 2011).

Figure 3 shows long and short positions taken by hedgers and speculators in the futures market, which are obtained from the CFTC futures-only Commitments of Traders (COT) report. Although participation in the futures market by hedgers and speculators has experienced steady growth from 1990 onwards, the increase in their positions has been faster after 2004. While both positions of hedgers and speculators increase over time, speculators take relatively more long positions than short positions. Since late 2007, positions taken by hedgers have gradually decreased while the speculators' positions continue to increase.

To measure the level of speculative activity, I use the speculation index (SI) which is designed to gauge the intensity of speculation relative to hedging (Working, 1960; Buyuksahin and Robe, 2010; Buyuksahin and Harris, 2011; Pan, 2011). If we denote SS (SL) as the speculator's short (long) positions, and HS (HL) as the hedger's short (long) position, the speculation index is defined as

$$SI_t = \begin{cases} 1 + \frac{SS_t}{HL_t + HS_t}, & if \;\; HS_t \geq HL_t, \\ 1 + \frac{SL_t}{HL_t + HS_t}, & if \;\; HS_t < HL_t \end{cases} \tag{4.1}$$

The speculation index measures the extent by which speculative positions exceed the necessary level to offset hedging position. For instance, a 1.1 speculation index means that there are 10% more speculative positions than what is needed to offset the hedging demand.

Figure 4 plots the speculation index defined in equation (4.1) from 1990 to 2008. This time series reveals that there has been a high level of speculation in recent years. Before 2002, the speculation index was around 1.05; however it has risen steadily over time to 1.16 in 2008. It suggests that speculative activities in excess of hedging needs in the crude oil market have increased since 2002.

5 Results

Starting with the risk-neutral density in 5.1, I estimate the market risk aversion using the probability density forecast methodology for the whole sample. With the market risk aversion estimates, I infer risk premiums from the physical densities and the risk-neutral densities in 5.2. In 5.3, I estimate the state-dependent market risk aversion and infer state-dependent market risk premiums. I interpret the empirical findings by relating them to the model implications in Section 2, and explain the impact of the increased financialization on the risk aversion and risk premiums in 5.4. In 5.5, I compare the predictability of state-dependent risk premiums.

5.1 Risk Neutral Density

I first fit Black (1976) implied volatilities for the cross-sectional option data on a given day, using a second order polynomial function of strike price and maturity, following Pan (2011). Then I construct a grid of strike prices and obtain at-the-money (ATM) Black (1976) implied volatilities from the fitted polynomial function for each maturity. With these implied volatilities, I back out call prices $\widehat{C}(F_{t,T}, K, t, T, \sigma(K,T))$ on the desired grid of strike prices, and then calculate the risk neutral density in equation (3.5) for the futures price at the maturity date T.

I define the futures return as $R_{t,T} = \log\left(\frac{F_T}{F_{t,T}}\right)$, where $F_{t,T}$ is the time t price of a futures contract maturing at time T, and $F_T = F_{T,T} \equiv S_T$.

The density function of futures returns over the period of $T - t$ is

$$\widehat{f}^Q(R_{t,T}|F_t) = \widehat{f}^Q(S_t \exp(u)|F_t) \times S_t \exp(u) \tag{5.1}$$

Figure 5 plots the risk-neutral density functions of futures returns maturing in four weeks, which are calculated from options prices according to equations (3.5) and (5.1). The risk-neutral density functions in Figure 5 suggest a strong pattern of stochastic volatility. The variances in the risk neutral density function path are larger for the crisis periods, such as late

1990 to early 1991, September 2001, March 2003, and the second half of 2008. These patterns are consistent with the implied volatility patterns plotted in Figure 2.

5.2 Constant Market Risk Aversion and Risk Premium

Using the risk neutral probability densities, I test the corresponding forecast ability by running the Berkowitz (2001) likelihood ratio (LR) test, as in equation (3.9). The p-values of the Berkowitz (2001) LR statistic and the estimates of μ, ρ, σ in (3.8) and (3.9) for different horizons are reported in Table 2, Panel A. The p-values reported in Table 2, Panel A suggest that for two weeks and six weeks horizons, I can reject the hypothesis that the risk neutral PDFs provide accurate forecasts of the futures distribution.

Assuming an initial value of Γ (for the power utility function) or η (for the exponential utility function), I then maximize the forecast ability of the resulting subjective probability density functions in equation (3.12) or (3.15) by maximizing the p-values of the Berkowitz LR statistic with respect to Γ (or η). This procedure gives the estimate of the Γ (or η).

Panel B and Panel C in Table 2 report the Berkowitz LR statistic p-values and the estimates of the market risk aversion coefficients from power-utility and exponential-utility adjusted density functions. The p-values of the Berkowitz LR test of physical densities are all much higher than that of the risk neutral density functions. It means the risk adjusted physical density functions have better forecast ability than risk neutral probability density functions.

For forecast horizons of one to five weeks, all the physical probability density functions have significant forecast ability. For the six week forecast horizon, both p-values of the Berkowitz LR test implied by the power utility function and the exponential utility function are lower than 5%, suggesting that both of them reject the null hypothesis. Note that rejection of the Berkowitz LR test does not necessarily suggest that they cannot provide an accurate density forecast. This finding may be true to the overlapping forecast period and violation of the independence hypothesis. This could be verified by the large correlation coefficient, ρ. For example, the estimated correlation coefficient of the physical probability density function implied by the power utility function is 0.33, which is different from the null hypothesis $\rho = 0$. Similar properties hold for risk neutral density functions and physical density functions implied by the exponential utility.

For forecast horizons of one to five weeks, the estimated market relative risk aversions implied by the power utility function are 2.03, 1.18, 1.39, 1.19, and 0.88, respectively. The estimates in the case of exponential utility are absolute risk aversions. For this case, the means of market relative risk aversions are also calculated and reported at the bottom of Table 2. They are 1.54, 0.88, 1.26, 0.99, and 0.67, respectively for the one to five weeks forecast horizons. These numbers, on average, are lower than most of the relative risk aversion coefficients documented in the literature on equity index options.[5] There are several possible explanations for this finding: one is that commercial hedgers have crude oil commodity as a natural hedge, thus they are relatively less risk averse than the investors in the equity market; another possible explanation is that financial investors who long crude oil futures for the purpose of portfolio diversification may require less premium to compensate their risk exposure in the crude oil market.

The process of maximizing the Berkowitz statistic with respect to the market risk aversion does not indicate whether the resulting risk aversion coefficient is significantly different from zero. The process of searching for the optimal level of Γ (or η) alters the distribution of the test statistic, biasing the likelihood ratio toward unity and thus overstating the p-value. To investigate the properties of the estimation procedure and the significance of the resulting estimates, I run the bootstrap test to check the distribution of the estimates, following Bliss and Panigirtzoglou (2004).

The bootstrap test captures the impact of the actual data and potential model misspecification on the reliability of parameter estimates. I apply the bootstrap test for each option horizon with M=1,000 replications. Each replication consists of drawing with replacement a random sample of pairs of densities and associated outcomes from the original sample. Each bootstrap sample was then used to estimate the risk aversion coefficient and the p-value of the Berkowitz LR statistic.

Since bootstrapping invalidates the independence assumption underlying the computation of p-values, the distribution of bootstrap p-values is uninformative. However, the distribution of risk aversion coefficients provides an indication of the sampling variation of these estimates.

Figure 6 gives the box plot of distributions of market risk aversion coefficients from bootstrap tests. Figure 6.A plots the market relative risk aversion coefficients implied by the power utility function and Figure 6.B plots the market absolute risk aversion coefficients implied by the exponential utility function. From these figures, I conclude that market risk aversion estimates are significantly different from zero across all forecast horizons and for both power and exponential utilities.

Table 3 reports bootstrap estimation statistics. Consistent with the results in Figure 6, the estimated market risk aversion coefficients are significantly different from zero for all forecast horizons.

Figure 7 plots the risk premiums calculated using function (5.2) for both the power utility function and the exponential utility function for futures options expiring in four weeks. We observe large spikes in the risk premium paths in late 1990 and 2008 for both utility functions.

[5] For example, Bliss and Panigirtzoglou (2004) find that the relative risk aversion implied by a power utility function using S&P 500 data is between 3.37 to 9.52; Guo and Whitelaw (2001) estimate a coefficient of relative risk aversion of 3.52; Ait-Sahalia and Lo (2000) report a value of 12.7.

For the risk premium implied by the power utility function, we also observe spikes around late 1998, 2001, and 2003. For example, the risk premium implied by the power utility function for the four-week forecast horizon in August 1990 is approximately 45% per year, while the risk premium implied by the exponential utility function for the four-week forecast horizon in December 2008 is more than 90% per year. These high risk premium periods coincide with the periods of high volatility in the crude oil market, if we compare Figure 7 with Figure 3 and Figure 5. The risk premiums inferred using constant risk aversion are positively correlated with market volatility.

5.3 State-Dependent Market Risk Aversion and Risk Premiums

To estimate state-dependent market risk aversion, it is necessary to identify states in the crude oil market. Hamilton and Wu (2011) and Kang and Pan (2011) investigate the role of risk premiums in the crude oil market and suggest the increased participation by speculators in the crude oil futures market may have been a factor in changing the nature of risk premium. Since the speculation index introduced in equation (4.1) measures the extent to which the speculative positions exceed the level needed to offset hedging positions, I choose this index as the state variable.

Using the speculation index as a criterion, I first distinguish two subsamples: one is the low-speculation period and the other is the high-speculation period. Each period has the same number of observations. For each subsample period, I run the Berkowitz (2001) density function forecast ability test of the underlying asset prices. The estimated market risk aversion coefficients and the Berkowitz LR p-values are reported in the first two rows in Table 4.

Speculation has been rising since 2002 (Figure 4). I therefore perform the empirical analysis using another two subsamples: January 1990 to December 2002, and January 2003 to December 2008. To compare our results with those of Hamilton and Wu (2011), I also analyze the other two subsamples by dividing the sample into the January 1990 to December 2004 and January 2005 to December 2008.

Table 4 presents the subsample estimation results for market risk aversion coefficients for the four-week forecast horizon. Physical probability density functions are implied using a power utility function. I also report the mean volatility level and mean speculation index level for each subsample period. The high speculation period corresponds to the high volatility period. For all three tests reported in Table 4, the market relative risk aversion coefficients estimated for the high speculation period are all lower compared to those estimated for low speculation periods. This confirms our hypothesis that as the speculative level increases, the market risk aversion level decreases.

Using the estimation results in Table 4, Figure 8 plots the subsample risk premiums for the period of January 1990 to December 2004 and January 2005 to December 2008. Compare to Figure 7, this figure shows sharp decreases in the level of risk premium after 2005. The risk premiums after 2005 are much less volatile and negative since the market risk aversion coefficient for the period of January 2005 to December 2008 is negative.

The subsample estimation in Table 4 divides the whole sample into two periods and estimates the market risk aversion coefficient for two sub-periods. This approach may not suffice to capture the state dependence of market risk aversion and risk premiums. I therefore estimate the state-dependent risk aversion coefficients defined in equations (3.16) and (3.17) for the power utility function and exponential utility function respectively. I run the Berkowitz (2001) forecast ability test by maximizing LR p-values for the risk adjusted physical density functions with respect to the values of coefficients a and b.

Refer to the level of speculation index as the state variable, x_t. Table 5 reports the estimation results of the coefficients a and b that determine state-dependent market risk aversion, and the corresponding p-values. Using the intuition provided by the model in Section 2, we expect the slope coefficient to be negative. The result from both the power utility function and the exponential utility function shows that the b estimates are significantly negative and confirms the negative relationship between market risk aversion and the level of the speculation index.

The state-dependent market relative risk aversion coefficients are plotted in Figure 9. From this figure, we observe that there are dramatic changes since 2002. The market risk aversion coefficient is relatively stable before 2002. After 2002, the market risk aversion coefficients are much more volatile, especially in the case of exponential utility. After 2005, relative risk aversion coefficients are more volatile and significantly smaller. For the period after 2007, the implied market risk aversion coefficients are negative most of the time using both power utility or exponential utility functions.

The state-dependent risk premiums calculated using state-dependent market risk aversion are plotted in Figure 10. Compared to Figure 7 and Figure 8, we see more dramatic variability in risk premiums over time when allowing for state dependence, especially in the case of exponential utility. The pattern of state-dependent risk premiums before 2002 is fairly similar to that calculated using a constant market risk aversion coefficient, except that the former risk premiums are larger in magnitude. After 2002, the state-dependent risk premiums on average are lower than in the first half of the sample. For some periods, such as late 2005 and late 2008, risk premiums are negative. This result is consistent with the empirical findings in Hamilton and Wu (2011).

5.4 Interpretation of State-Dependent Market Risk Aversion and Risk Premiums

It is worth noting that when assuming state-independent MRA in Table 4, the estimated market relative risk aversion for the period of 2005-2008 in Table 4 is negative. Assuming state-dependent market relative risk aversion, the estimated market relative risk aversion after 2007 is negative most of the time. Negative risk aversion might be counter-intuitive. However, in a commodity market with increased financialization, negative risk aversion may not be unreasonable. I interpret these findings by relating them to the stylized model in Section 2.

The implications in Section 2.3 are based on the premise that traders in the crude oil market are risk averse and crude oil futures contracts are initiated by commercial traders for hedging purposes. These assumptions are in line with the assumptions underlying Keynes' theory of normal backwardation. Producers of the physical commodity want to hedge their price risk by selling futures contracts. They want to offer a risk premium for this privilege, and speculators will be compensated for insuring the commercial hedgers.

However, the commodity market has been experiencing dramatic structural changes over the last decade. Many researchers ascribe them to the increased financialization of the commodity market. The financial industry has developed new instruments that allow institutions and individuals to invest in commodities, for example, long-only commodity index funds (LOCF), over the counter (OTC) swap agreements, exchange traded funds (ETF) and other products (Irwin and Sanders, 2011). These instruments provide investors with buy-side exposure from a particular index of commodities. Several studies have concluded that investors can reduce portfolio risks through investments in long-only commodity index funds (LOCF) (Gorton and Rouwenhorst, 2006; Erb and Harvey, 2006).

If a speculator invests in crude oil futures for the purpose of portfolio diversification, she may choose long positions regardless of what happens to fundamentals and no matter what the crude oil spot position is. The more long positions a speculator holds, the less risk averse she is, and the lower the aggregate market risk aversion. To insure her long position, the speculator would like to offer, instead of receiving, a risk premium to her counterparty. Therefore she requires less premium to compensate her risk exposure in the crude oil market.

As in equation (2.12), before financialization, speculators take positions to offset the hedger's position. $\pi_{h,t}$ and $\theta_{s,t}$ should have the same signs and the risk aversion coefficient of the speculator (γ_t^s) should be positive. When the financial speculator starts to take excess speculative positions for the purpose of portfolio diversification, she would long more futures contracts than the necessary level to offset the hedger's position, ($\theta_{s,t} > 0$ and $\theta_{s,t}$ increases). The speculator's risk aversion coefficient (γ_t^s) decreases, and it reduces market risk aversion

(Γ_t) accordingly. As the financial speculator's long positions keep increasing, the risk aversion coefficient of the speculator (γ_t^s) could be negative. In some extreme cases, if a speculator longs futures contracts ($\theta_{s,t} > 0$) when the aggregate spot position is in net demand ($\pi_{h,t} < 0$), the aggregate risk aversion coefficient would be negative too ($\gamma_t^s < 0$ and $\Gamma_t < 0$).[6]

A negative risk aversion coefficient does not necessarily means the speculator is risk loving. A speculator who seeks portfolio diversification cares the overall profit of her portfolio and therefore possibly has a higher risk tolerance in the crude oil futures market. I interpret the negative risk aversion coefficient as a signal of excess buying pressure from investors seeking portfolio diversification, even if the physical commodity is in net demand. This excess buying pressure in the futures market could shift the beneficiary of the risk premiums from the long side of the futures contract to the short side. Hamilton and Wu (2011) also document significant changes in oil futures risk premiums since 2005. Compensation to the long position is smaller on average but much more volatile, and often significantly negative when the futures curve slopes upward. This observation is consistent with the claim that index-fund investing has become more important relative to commercial hedging in determining the structure of crude oil futures risk premium over time.

To summarize, the negative values of market risk aversion may imply that speculators took more excess long positions and applied large buying pressure to the crude oil futures market. Speculators who care about portfolio diversification would want to long the crude oil futures and pay, instead of receiving, a risk premium for their position in some extreme cases.

5.5 Predictive Power of the State-Dependent Risk Premiums

Since the risk premiums are obtained by maximizing the forecast ability of the probability density function of the crude oil futures prices, one would expect that they have explanatory power for predicting crude oil futures returns. In state-dependent case, one would expect the risk premiums to have high predictive power because of the increased flexibility.

To test the predictive power of the state-dependent market risk premiums, I regress futures returns on the state-dependent market risk premiums. For comparison, I also run regressions of futures returns on risk premiums calculated using constant risk aversion coefficients, as well as several other predictors used in the existing literature, such as the lagged futures returns and the lagged ATM volatilities.

I run the regression

[6] The simple stylized model in Section 2 which considers only the crude oil futures market cannot completely explain the risk preference of a speculator who has a diversified portfolio. However, the relationship between risk aversion and speculative futures positions should still hold.

$$\log\left(\frac{F_T}{F_{t+1,T}}\right) = c_i + d_i y_{i,t} + \varepsilon_{i,t+1} \tag{5.3}$$

where $y_{i,t}$ is the predictor i at time t, and c_i and d_i are parameters to be estimated.

Table 6 reports the regression coefficients in equation (5.3) and the R-squares of the regressions. Most of the independent variables, such as lagged ATM volatility, risk premius using the power utility or exponential utility, and laaged futures return, have very low R-squares and insignificant coefficient estimates. The lagged futures returns has an R-square of 1.52%, and the speculation index has an R-square of 1.63%. R-squares for lagged ATM volatility and risk premiums calculated using single constant market risk aversion are even lower. In allowing for state dependence in risk premiums, I improve the predictability of futures returns greatly. The R-square is 2.08% in the case of the state-dependent risk premium implied by power utility function, which it is 6.8% using the state-dependent risk premium implied by exponential utility. The d_i coefficients estimated using the state-dependent risk premiums implied by the power utility function and exponential utility function are both significant.

One may argue that these R-squares are small. Neely, Rapach, Tu, and Zhou (2010) discuss the magnitude of R-squares in predictive regressions and conclude that because monthly returns inherently contain a substantial unpredictable component, a monthly R-square of 0.5% represent economically significant predictive power. So the state-dependent risk premiums obtained in this research have significant explanatory power in predicting crude oil futures returns.

6 Robustness

To test the robustness of my results, I re-estimate the market risk aversion using an alternative methodology following Melick and Thomas (1997), Ritchey (1990), and Liu, Shackleton, Taylor, and Xu (2005).

These studies define the risk neutral density of the asset price at option expiration as a mixture of lognormal densities (hereafter MLN densities). The MLN density function, g_{MLN}, is the weighted combination of two lognormal densities, g_{LN}:

$$g_{MLN}(x|\theta) = w g_{LN}(x|F_1, \sigma_1, T) + (1-w) g_{LN}(x|F_2, \sigma_2, T) \tag{6.1}$$

with

$$g_{LN}(x|F, \sigma, T) = \frac{1}{x\sigma\sqrt{2\pi T}} \exp(-\frac{1}{2}\left[\frac{\log(x) - [\log(F) - 0.5\sigma^2 T]}{\sigma\sqrt{T}}\right]) \tag{6.2}$$

The parameter vector is $\theta = (F_1, F_2, \sigma_1, \sigma_2, w)$, with $1 \leq w \leq 1$ and $F_1, F_2, \sigma_1, \sigma_2 > 0$. The parameters F_1, σ_1 and w denote the mean, volatility and weight of the first lognormal density, while F_2, σ_2 and $1-w$ denote the mean, volatility and weight of the second lognormal density.

The density is risk neutral when its expectation equals the current futures price F, i.e.

$$wF_1 + (1-w)F_2 = F \tag{6.3}$$

The theoretical European option pricing formula is the weighted average of two option prices given by Black (1976) pricing formula for options on futures, denoted by $c_B(.)$

$$c(X|\theta, r, T) = wc_B(F_1, T, X, r, \sigma_1) + (1-w)c_B(F_2, T, X, r, \sigma_2) \tag{6.4}$$

Now consider the physical density $\widetilde{g}(x|\theta,\gamma) = g_{MLN}(x|\widetilde{\theta})$ with physical parameters $\widetilde{\theta} = (\widetilde{F}_1, \widetilde{F}_2, \sigma_1, \sigma_2,)$. Assuming a power utility function,

$$\widetilde{F}_i = F_i \exp(\gamma\sigma_i^2 T), \text{ for } i = 1, 2 \tag{6.5}$$

and

$$\frac{1}{\widetilde{w}} = 1 + \frac{1-w}{w}(\frac{F_2}{F_1})^{\gamma} \exp(\frac{1}{2}(\gamma^2-\gamma)(\sigma_2^2-\sigma_1^2)T) \tag{6.6}$$

The market risk aversion parameter, γ, is obtained using a two stage estimation technique. In the first stage, we estimate the RND parameter, θ, by minimizing the average squared difference between observed market option prices and theoretical option prices

$$\frac{1}{N}\sum_{i=1}^{N}(c_{market}(X_i) - c(X_i|\theta))^2 \tag{6.7}$$

where N is the number of European option prices used for a particular day, and $c(X|\theta)$ is the associated theroretical option pricing formula, given by equation (6.4). We run this estimation for $i = 1, 2, ..., n$ months and the estimated RND vectors $\widehat{\theta_i}$ vary across time.

In the second stage, we estimate the transformation parameter, γ, by maximizing the log likelihood function

$$\log(L(S_{T,1}, S_{T,2}, ...S_{T,n}|\gamma)) = \sum_{i=1}^{n}\log(\widetilde{g}_i(S_{T,i}|\widehat{\theta}_i, \gamma)) \tag{6.8}$$

Let $\{t_i, 1 \leq i \leq n\}$ be the n set of times when the chosen option contracts expire, $S_{T,i}$ denotes the observed price of the underlying asset at time t_i.

Table 7 reports the estimation results for options expiring in four weeks. As in Liu, Shackleton, Taylor, and Xu (2005), the physical density is calculated from the risk neutral density assming a CARA power utility function. In Panel A, the estimated market relative risk aversion coefficient for the entire sample (January 1990 - December 2008) is 1.8026. In Panel B, I estimate the market risk aversion coefficients for the subsamples used in Section 5. The market risk aversion for the first half of the sample are again higher than those for the second half of the sample. For example, the market risk aversion for the period of January 1990 to December 2002 is 2.0832 while that for January 2003 to December 2008 is 1.2085. These findings are consistent with the findings in Section 5.3 that as the speculation level increase, the market risk aversion decreases.

In Panel C, I estimate the state dependent market risk aversion coefficients as defined in equation (3.16). Refering to the level of speculation index as the state variable, I find that b is negative. These findings are again similar to those in Section 5.3: there is a negative relationship between market risk aversion and the level of speculation. Figure 11 plots the time-varying market relative risk aversion, and it is clear that after 2002 when the speculative activity in the crude oil market increases, the market risk aversion decreases dramatically. This relationship between market risk aversion and the level of speculation index is quite robust across different estimation methods.

7 Conclusion

I investigate if speculative activity in the crude oil market affects market risk aversion and risk premiums. Using WTI crude oil futures and option data, I estimate time-varying market risk aversion by specifying the speculation level as a state variable. I find that as the speculation level increases, market risk aversion and risk premiums decrease.

When allowing for state dependence in the risk aversion coefficient, the implied risk premiums change significantly over time: they are higher during the first half of the sample period, while after 2002, when speculation increases, they are on average smaller in magnitude and more volatile. This finding is consistent with that in Hamilton and Wu (2011) and suggests that speculators who invest in the crude oil market for the purpose of portfolio diversification are willing to accept lower risk premiums for their speculative positions.

Risk premiums implied by state-dependent market risk aversion have significantly higher explanatory power in predicting subsequent futures returns, compared with several commonly used predictive variables, especially when we assume exponential utility.

Overall my findings indicate that speculative activity in the crude oil market has had a significant effect on aggregate risk aversion and the evolution of risk premiums.

8 Appendix: Market Participants' Optimal Positions

I first solve for the hedger's optimal position. For the hedger's margin account at time t, $X_t^{\theta_{h,t}}$, I define

$$dX_t^{\theta_{h,t}} = \alpha_t ds + \beta_t dB_t \tag{A.1}$$

From equation (2.4), we have $\alpha_t = rX_t^{\theta_{h,t}} + m_t\theta_{h,u}^T F_t$ and $\beta_t = \theta_{h,u}^T F_t v_t$.

Let the dynamics of the hedger's wealth process, $W_t^{\theta_{h,t}}$, be

$$dW_t^{\theta_{h,t}} = a_t ds + b_t dB_t \tag{A.2}$$

where $\theta_{h,t}$ is the futures strategy of the hedger at time t. From equations (2.1) – (2.3) and (A.1), we have $a_t = rX_t^{\theta_{h,t}} + \pi_{h,t}\mu_t S_t + m_t\theta_{h,u}^T F_t$ and $b_t = \pi_{h,t}\sigma_t S_t + \theta_{h,u}^T F_t v_t$.

Assume an exponential utility function

$$U(W_t^{\theta_{h,t}}) = -\exp(-\gamma_t^h W_t^{\theta_{h,t}}) \tag{A.3}$$

where γ_t^h is the hedger's absolute risk aversion coefficient.

Let $Y_t^{\theta_{h,t}} = \exp(-\gamma_t^h W_t^{\theta_{h,t}})$. Apply Ito's Lemma

$$\begin{aligned} Y_{T-t}^{\theta_{h,t}} &= Y_0^{\theta_{h,t}} + \int_0^{T-t} Y_s^{\theta_{h,s}} \left[\frac{\gamma_s^{h^2}}{2} tr(b_s^T b_s) - \gamma_s^h a_s \right] ds \\ &\quad + \int_0^{T-t} Y_s^{\theta_{h,s}} \left[-s\gamma_s^h b_s \right] dB_s \end{aligned} \tag{A.4}$$

Thus,

$$E\left[Y_{T-t}^{\theta_{h,t}}\right] = Y_0^{\theta_{h,t}} + E\left[\int_0^{T-t} Y_s^{\theta_{h,s}} \left[\frac{\gamma_s^{h^2}}{2} tr(b_s^T b_s) - \gamma_s^h a_s \right] ds \right] \tag{A.5}$$

$\theta_{h,t}^*$ minimizes $\frac{\gamma_t^{h^2}}{2} tr(b_t^T b_t) - \gamma_t^h a_t$ at each time t. Taking the derivative, we get

$$\theta_{h,t}^* = -\frac{(v_t v_t')^{-1}}{F_t}[v_t\sigma_t'\pi_{h,t}S_t - m_t/\gamma_t^h] \tag{A.6}$$

A similar derivation can be obtained for the speculator, which gives equation (2.9).

References

[1] Ait-Sahalia, Y. and Lo, A. W., 2000. Nonparametric risk management and implied risk aversion. Journal of Econometrics 94, 9-51.

[2] Ait-Sahalia, Y., Wang, Y., and Yared, F., 2001. Do option markets correctly price the probabilities of movement of the underlying asset? Journal of Econometrics 102, 67-110

[3] Baker S. D., 2012. The financialization of storable commodities. Working paper, Carnegie Mellon University.

[4] Barillas, F. and Nimark K., 2012. Speculation, risk premia and expectations in the yield curve. Working paper, Emory University.

[5] Barone-Adesi, G. and Whaley, R., 1987. Efficient analytic approximation of American option values. Journal of Finance 42, 301-320.

[6] Berkowitz, J., 2001. Testing density forecasts with applications to risk management. Journal of Business and Economic Statistics 19, 465-474.

[7] Black, F., 1976. The pricing of commodity contracts. Journal of Financial Economics 3, 167-179.

[8] Bliss, R. R. and Panigirtzoglou, N., 2004. Option-implied risk aversion estimates. The Journal of Finance 1, 407-446.

[9] Breeden, D. T., 1984. Futures markets and commodity options: hedging and optimality in incomplete markets. Journal of Economic Theory 32, 275-300.

[10] Brigo, D. and Mercurio, F. 2002. Lognormal-mixture dynamics and calibration to market volatility smiles. International Journal of Theoretical and Applied Finance 5, 427-446.

[11] Buyuksahin, B. and Harris, J., 2011. Do speculators drive crude oil futures prices? The Energy Journal 32, 167-202.

[12] Buyuksahin, B. and Robe, M., 2010. Speculators, commodities and cross-market linkages. Working Paper, U.S. Commodity Futures Trading Commission.

[13] Christoffersen, P., Heston, S., and Jacobs, K., 2013. Capturing option anomalies with a variance-dependent pricing kernel. The Review of Financial Studies 26 (8), 1963-2006.

[14] Cochrane, J. H. and Hansen, L. P., 1992. Asset pricing explorations for macroeconomics, in 1992 NBER Macroeconomics Annual (NBER, Cambridge, MA).

[15] Duffie, D., 2001. Dynamic asset pricing theory. Third Edition. Princeton, New Jersey: Princeton University Press, Princeton and Oxford.

[16] Duffie, D. and Jackson, M. O., 1990. Optimal hedging and equilibrium in a dynamic futures market. Journal of Economic Dynamics and Control 14, 21-33.

[17] Einloth, J. T., 2009. Speculation and recent volatility in the price of oil. Working Paper, Federal Deposit Insurance Corporation.

[18] Erb, C. B. and Harvey, C. R., 2006. The strategic and tactical value of commodity futures. Financial Analysts Journal 62 (2), 69-97.

[19] Gorton, G. B. and Rouwenhorst, K. G., 2006. Facts and fantasies about commodity futures. Financial Analysts Journal 62 (2), 47-68.

[20] Greer, R., 2000. The nature of commodity index returns. Journal of Alternative Investment 3 (1), 45-52.

[21] Guo, H. and Whitelaw, R., 2001. Risk and return: some new evidence. Working Paper, Federal Reserve Bank of St. Louis.

[22] Hamilton, J. D. and Wu, J. C., 2012. Risk premia in crude oil futures prices. Working paper, University of California, San Diego.

[23] Hong, H. and Yogo, M., 2012. What does futures market interest tell us about the macroeconomy and asset prices? Journal of Financial Economics 105 (3), 473-490.

[24] Irwin, S. and Sanders, D., 2011. Index funds, financialization, and commodity futures markets. Applied Economic Perspectives and Policy 33 (1), 1-31.

[25] Jackwerth, J. C., 2000. Recovering risk aversion from option prices and realized returns. Review of Financial Studies 13, 433-467.

[26] Jorion, P. and Giovannini, A., 1993. Time series test of a non-expected utility model of asset pricing. European Economic Review 37, 1083-1100.

[27] Keynes, J. M., 1930. A treaties on money. Vol II, Macmillan, London.

[28] Litzenberger, R. H. and Rabinowitz, N., 1995. Backwardation in oil futures markets: theory and empirical evidence. Journal of Finance 50, 1517-1545.

[29] Liu, X., Shackleton, M. B., Taylor, S. J., and Xu Xinzhong. 2005. Closed-form Transformations from risk-neutral to real-world distributions. Journal of Banking and Finance 31, 1501-1520.

[30] Mehra, R. and Prescott, E. C., 1985. The equity premium: a puzzle. Journal of Monetary Economics 15, 145-161.

[31] Melick, W. R. and Thomas, C. P. 1997. Recovering an asset's implied PDF from option prices: an application to crude oil during the Gulf crisis. Journal of Financial and Quantitative Analysis 32, 91-115.

[32] Neely, C. J., Rapach, D. E., Tu, J., and Zhou, G., 2010. Forecasting the equity risk premium: the role of technical indicators. Management Science, forthcoming.

[33] Ng., V. K. and Pirrong, S. C., 1996. Price dynamics in refined petroleum spot and futures markets. Journal of Empirical Finance 2, 359-388.

[34] Pan, X., 2011. Investor beliefs and state price densities in the crude oil market. Working Paper, Tulane University.

[35] Ritchey, R. J. 1990. Call option valuation for discrete normal mixtures. Journal of Financial Research 13, 285-296.

[36] Rosenberg, J. V. and Engle, R. F., 2002. Empirical pricing kernels. Journal of Financial Economics 64, 341-372.

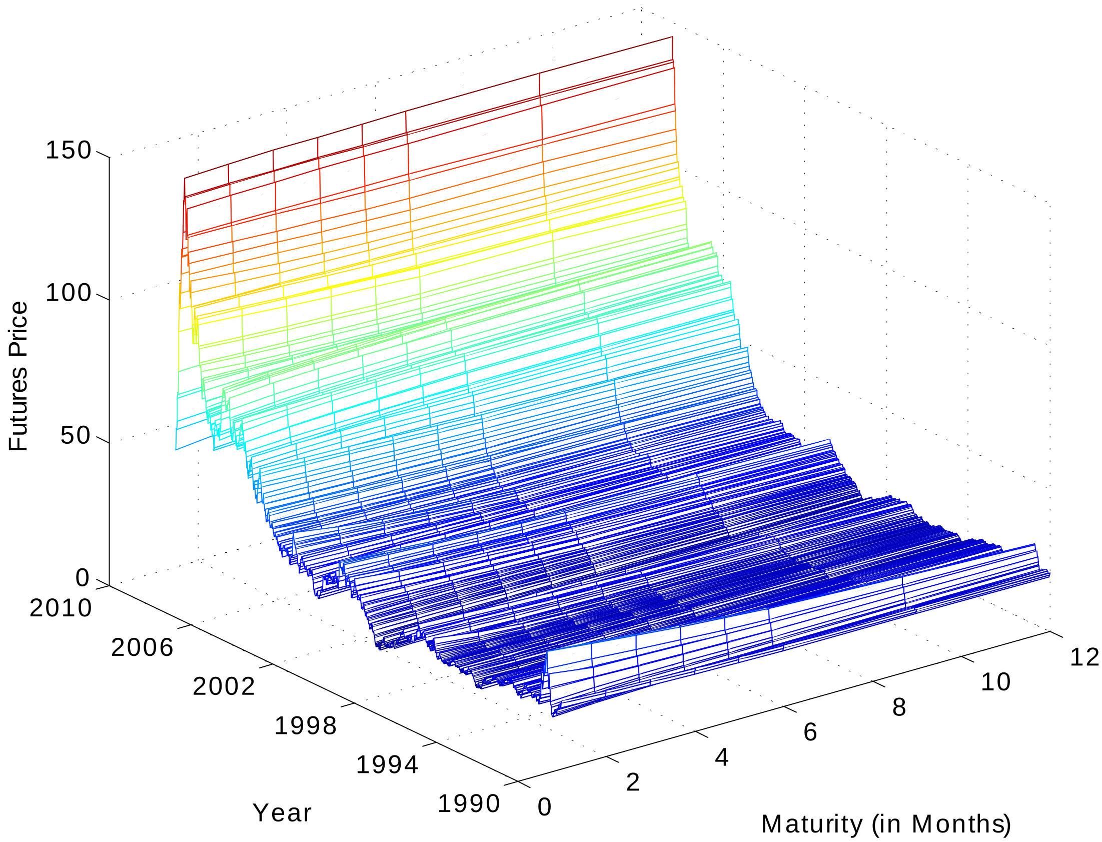
[37] Singleton, K. J., 2011. Investor flows and the 2008 boom/bust in oil prices. Working paper, Stanford University.

[38] Tang, K. and Xiong W., 2012. Index investment and the financialization of commodities. Financial Analysts Journal 68, 54-74.

[39] Trolle, A. B. and Schwartz, E. S. 2009. Unspanned stochastic volatility and the pricing of commodity derivatives. Review of Financial Studies 22, 4423-4461.

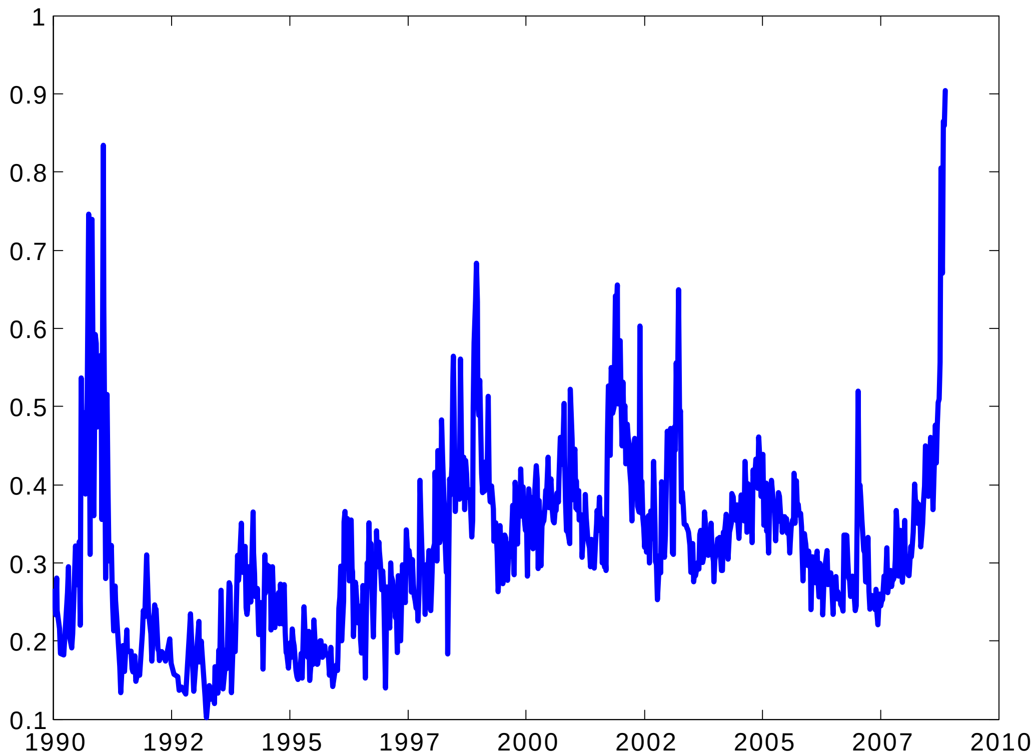
[40] Working, H. 1960. Speculation on hedging markets. Stanford University Food Research Institute Studies 1, 185-220.

Figure 1: Futures Prices



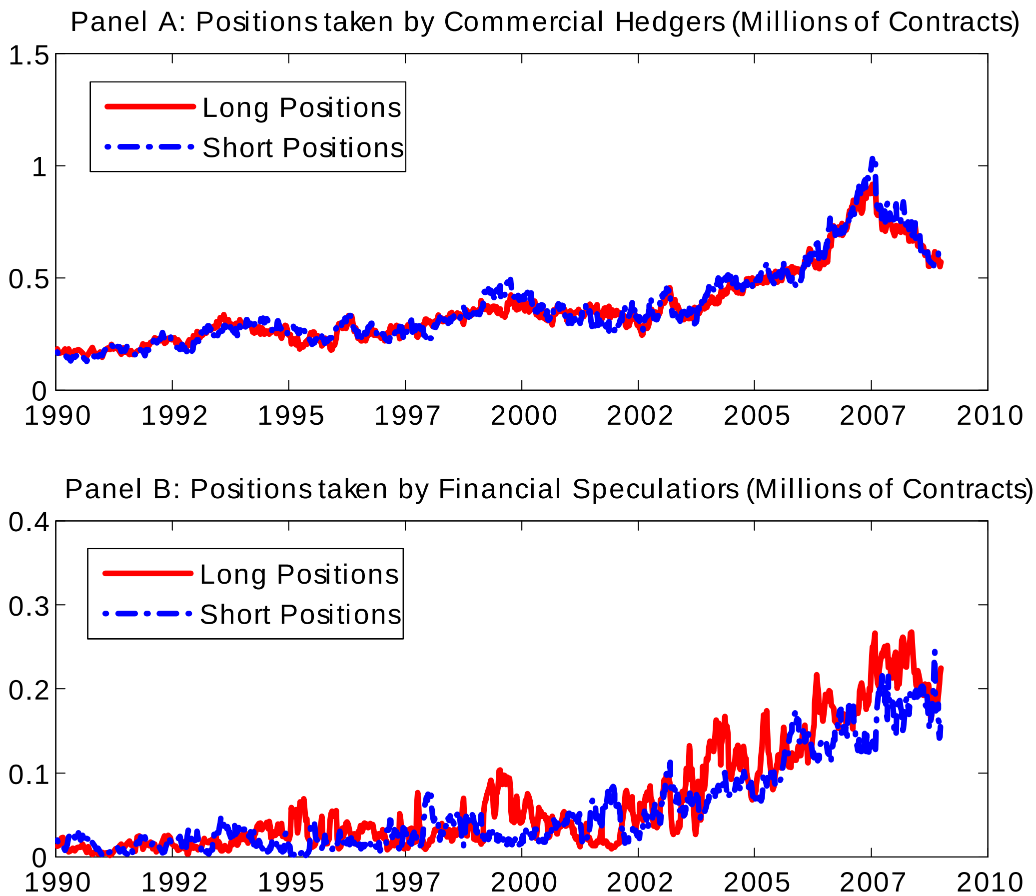
Notes to Figure: This figure plots the prices of futures contracts maturing in 1, 2, 3, 4, 5, 6, 9, and 12 months (M1-M2 and Q1-Q2 futures contracts). The data spans 4,753 trading days from January 2, 1990 to December 3, 2008. To avoid cluttering the figure, only the futures term structures for Wednesdays are displayed.

Figure 2: Implied Volatility of ATM Futures Options with One-Month Maturity



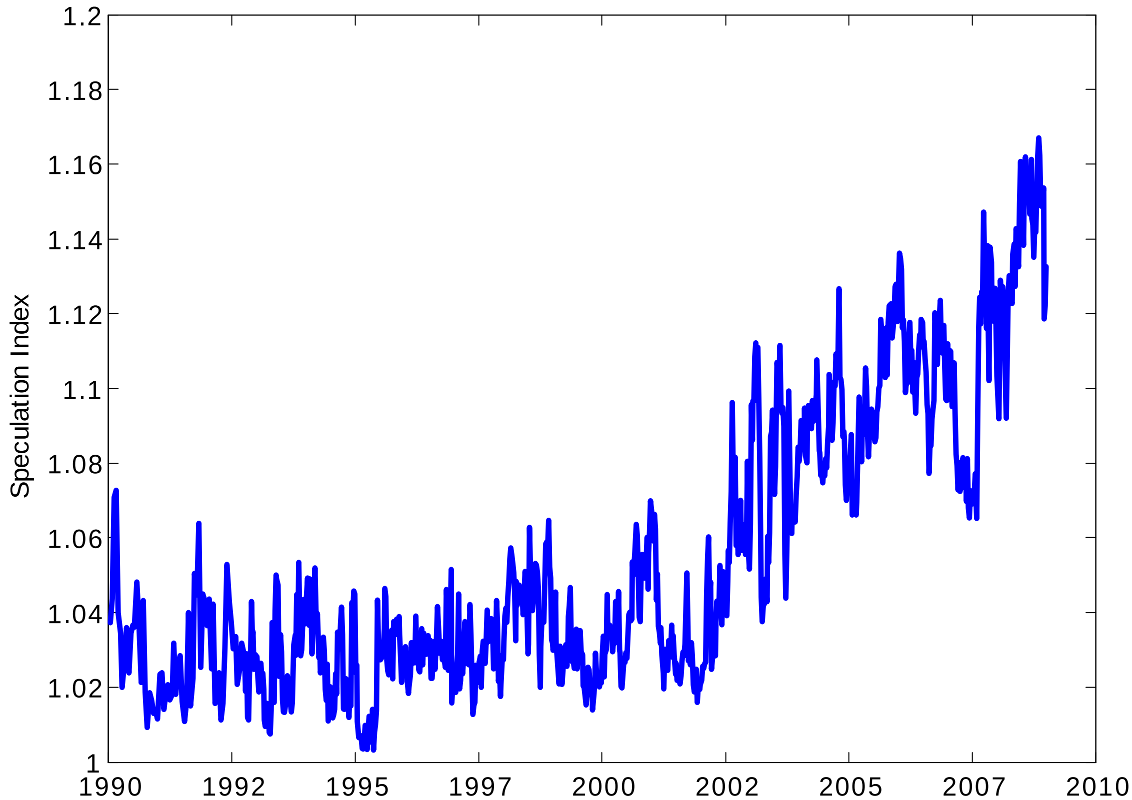
Notes to Figure: This figure plots the implied volatility of filtered ATM options on futures contracts maturing in 1 month. The data spans 4,753 trading days from January 2, 1990 to December 3, 2008.

Figure 3: Traders' Futures Positions



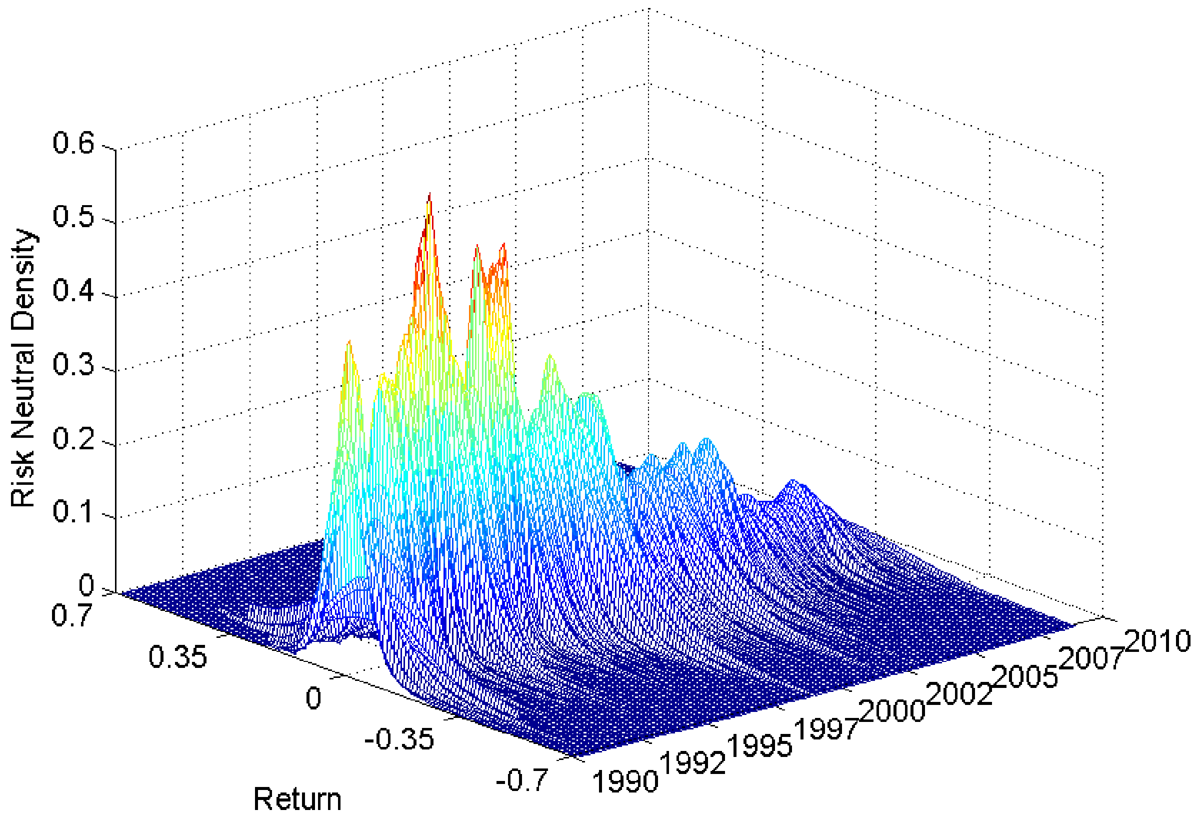
Notes to Figure: This figure plots futures positions taken by commercial hedgers and financial speculators from January 1990 to December 2008. Positions of hedgers and speculators are from the U. S. Commodity Futures Trading Commission's (CFTC) futures only Commitments of Traders (COT) report.

Figure 4: Speculation Index



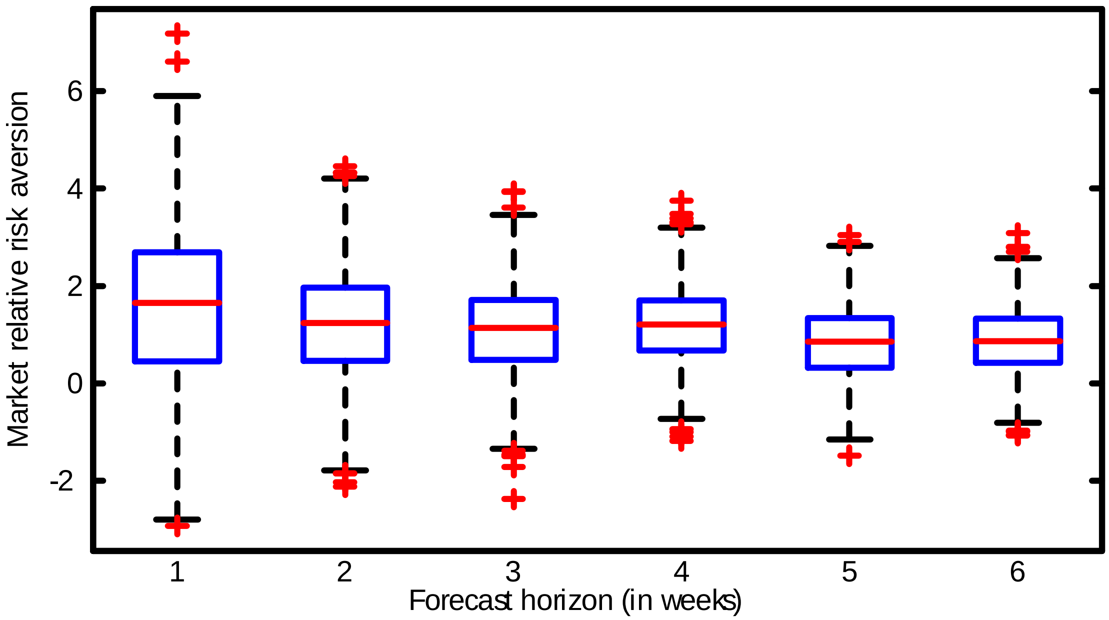
Notes to Figure: This figure plots the speculation index defined in equation (4.1).

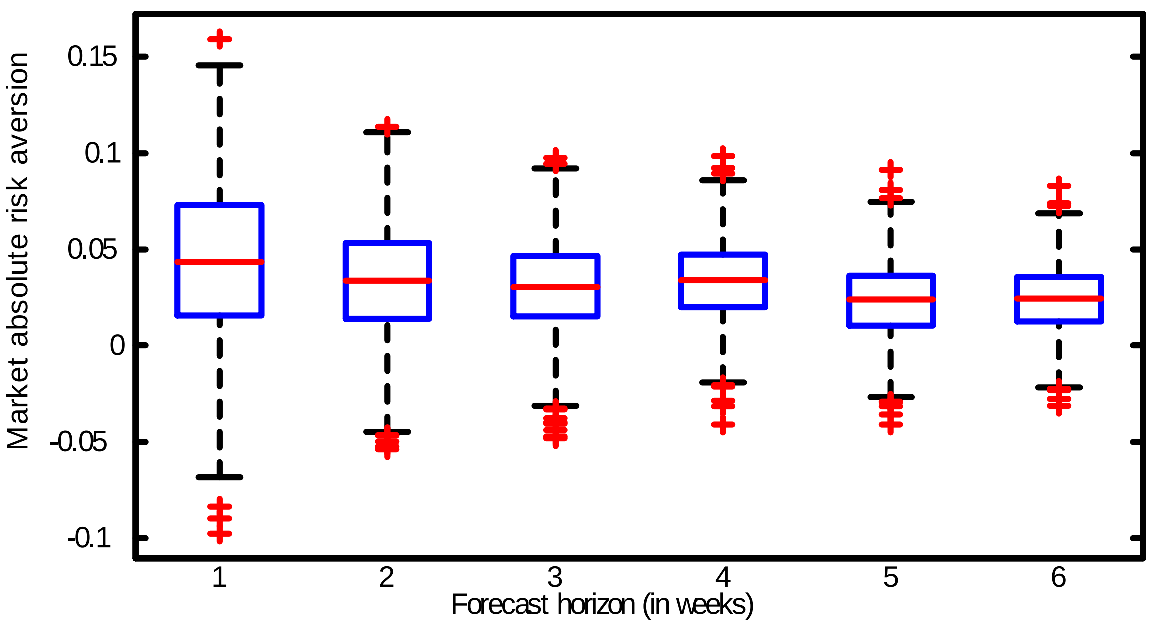
Figure 5: Risk Neutral Probability Density Functions of Futures Returns with Maturity of Four Weeks



Notes to Figure: This figure plots the risk neutral density functions of futures returns maturing in four weeks, which are calculated from options prices according to equations (3.5) and (5.1).

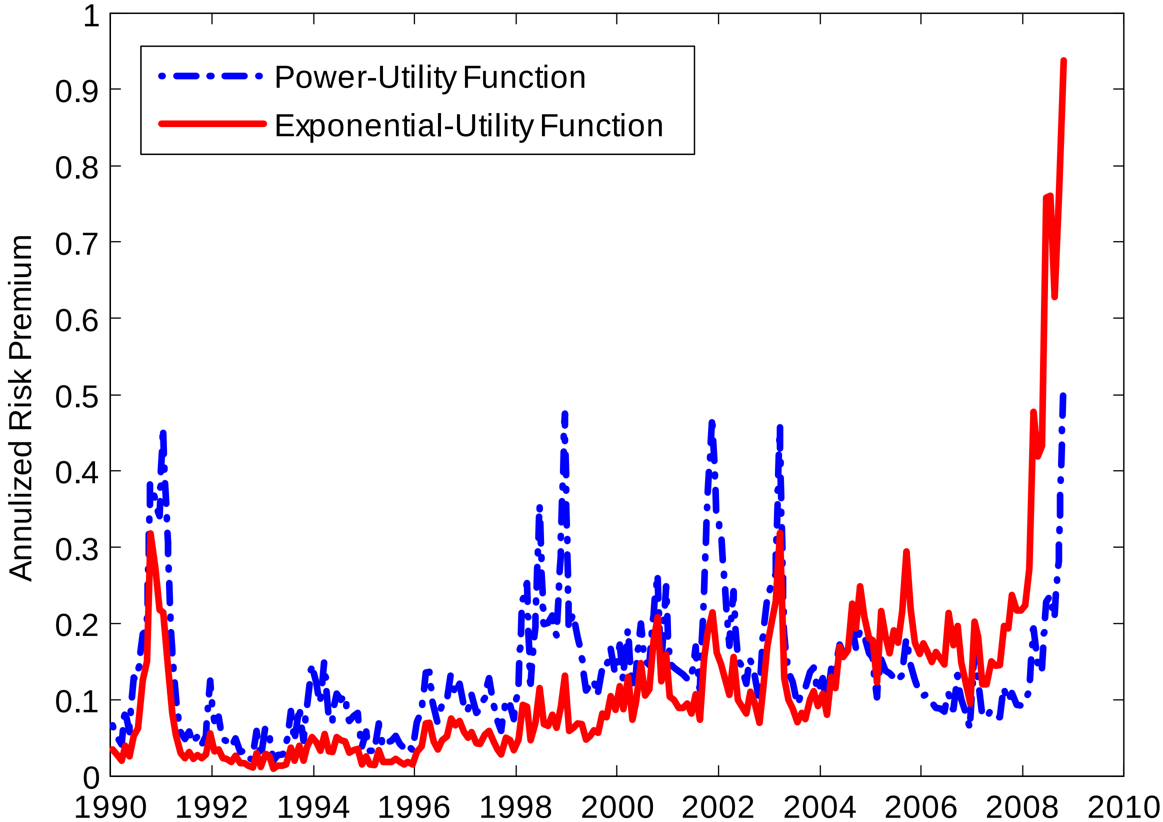
Figure 6: Bootstrapped Distribution of Market Risk Aversion





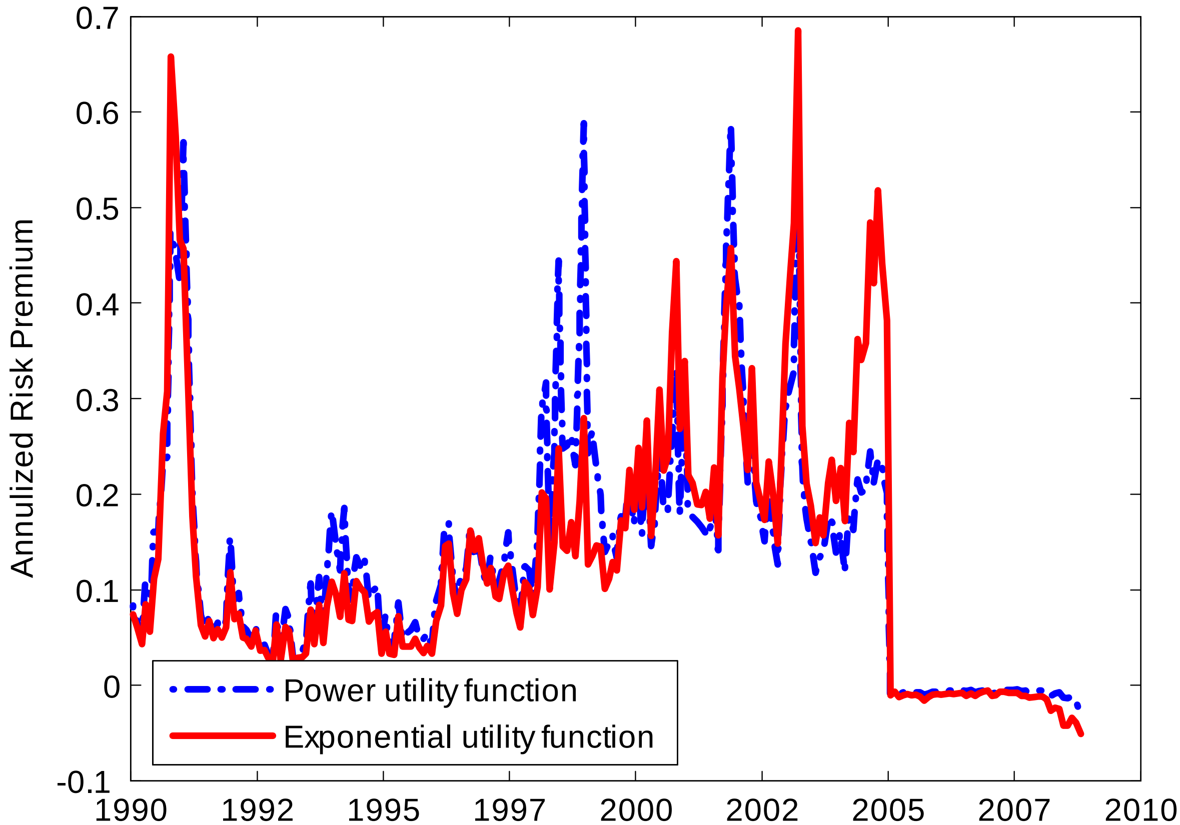
Notes to Figure: These figures plot bootstrapped distributions of market risk aversion coefficients. Panel 6.A plots the market relative risk aversion coefficients implied by the power utility function and Panel 6.B plots the market absolute risk aversion coefficients implied by the exponential utility function.

Figure 7: Implied Risk Premiums with Constant Market Risk Aversion



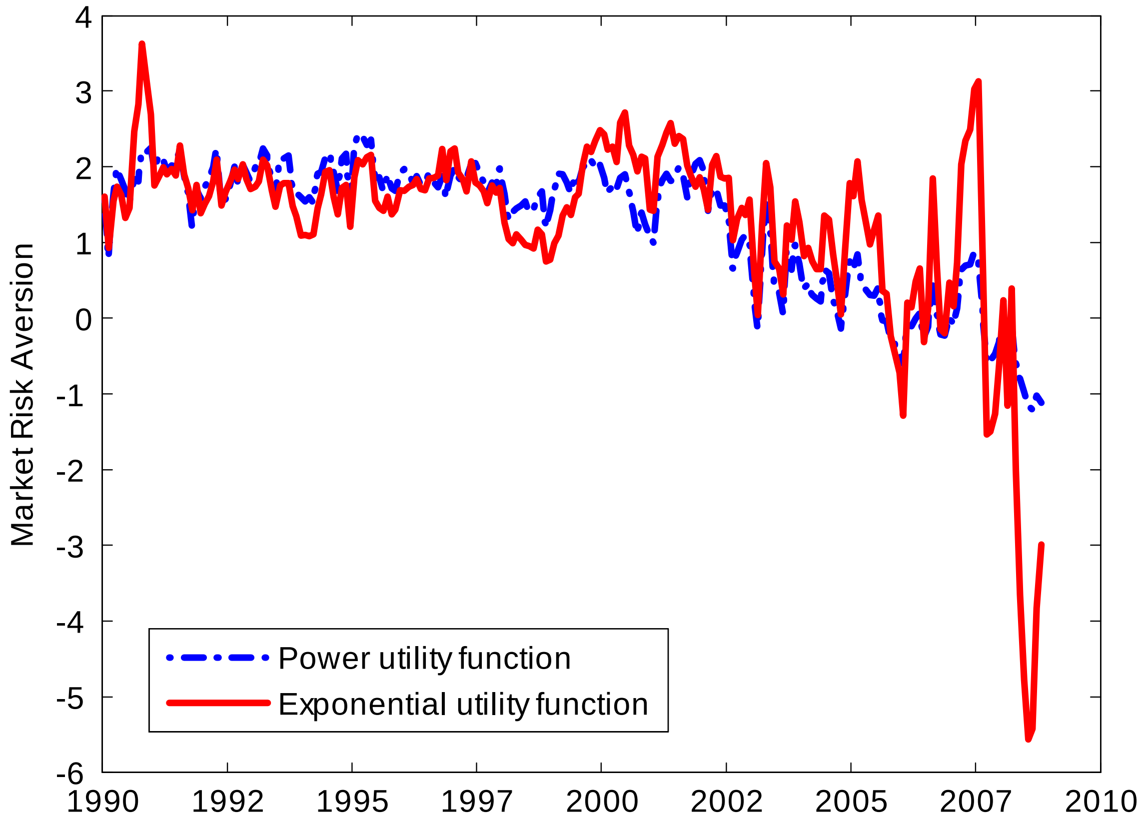
Notes to Figure: This figure plots the risk premiums in the crude oil market calculated using equation (5.2).

Figure 8: Risk Premiums Using Subsample Estimates



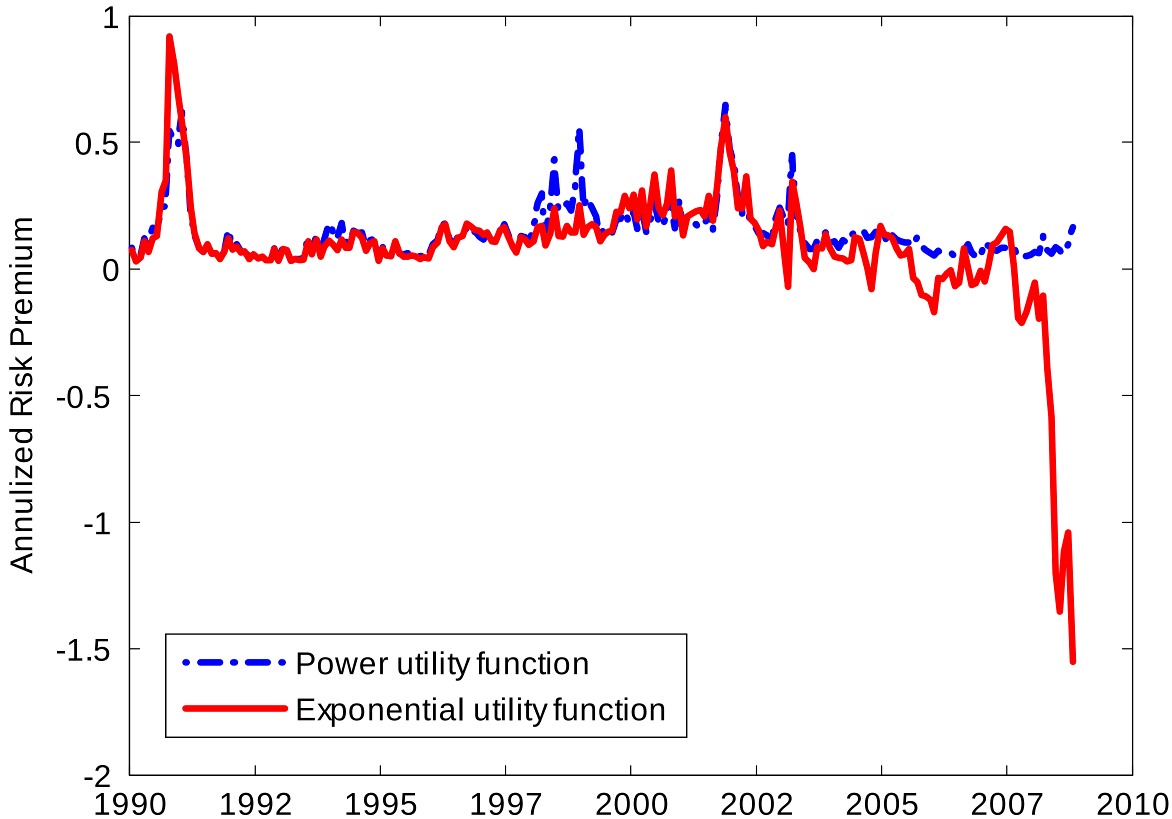
Notes to Figure: This figure plots risk premiums obtained using separate estimation for the periods 1990-2004 and 2005-2008. The parameter estimates are from Table 4.

Figure 9: State-Dependent Market Risk Aversion Coefficients



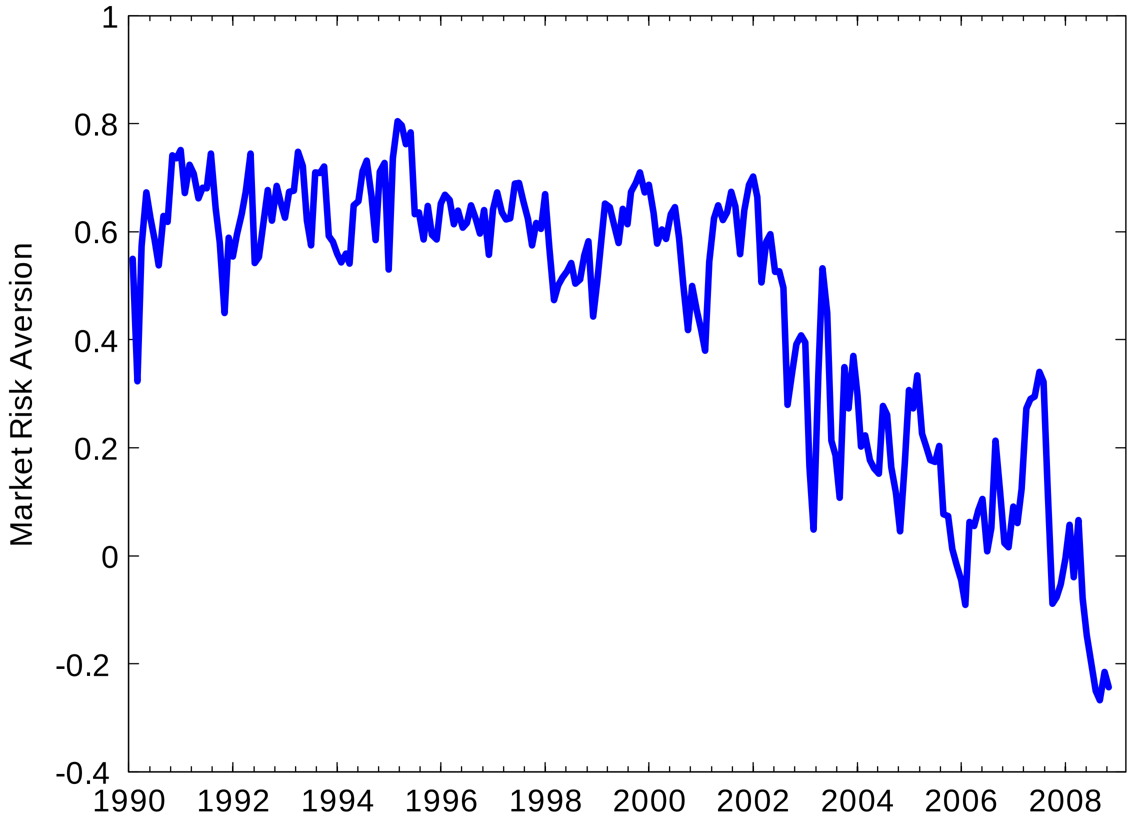
Notes to Figure: This figure plots the state-dependent market risk aversion coefficients calculated using equation (5.3). The parameter estimates are from Table 5.

Figure 10: State-Dependent Risk Premiums



Notes to Figure: This figure plots the state-dependent risk premiums calculated using the state-dependent market risk aversion in Figure 9.

Figure 11: Robustness Check: State-Dependent Market Relative Risk Aversion



Notes to Figure: This figure plots the state-dependent risk aversion coefficients calculated using the estimation results in Table 7 Panel C, following the methodology in Liu, Shackleton, Taylor, and Xu (2005).

Table 1: Crude Oil Option Data Summary Statistics

Forecast Horizon	**1w**	**2w**	**3w**	**4w**	**5w**	**6w**	**2m**	**3m**	**4m**	**5m**	**6m**	**7m**	**8m**	**9m**	**1y**
No. of Cross-sections	180	212	220	226	225	226	218	215	221	215	177	173	156	135	148
No. of Contracts / Cross-section	17.19	22.82	26.84	29.81	31.13	32.34	29.45	27.59	25.46	21.05	3706	3129	2331	1769	15443
Implied Volatility	0.41	0.4	0.4	0.4	0.39	0.39	0.37	0.34	0.33	0.32	0.3	0.29	0.28	0.27	0.3
Option Price	0.29	0.4	0.49	0.57	0.65	0.71	1.38	1.61	1.81	1.82	1.97	2.07	2.13	2.3	1.99
Strike Price	55.35	55.69	55.62	54.74	53.18	52.46	52.54	47.76	48.34	44.73	43.81	44.5	42.08	41.07	42.96
Futures Price	55.02	54.78	54.83	53.9	53.05	52.55	52.98	49.52	51.18	47.54	46	47.23	44.85	43.43	45.37
Open Interest (OI)	5233	5471	5311	5011	4519	4144	3190	2323	2215	1740	1684	1779	1559	1422	1814
Volume	695	476	410	429	341	257	181	112	64	44	36	30	29	32	72
Volume / OI	0.13	0.09	0.08	0.09	0.08	0.06	0.06	0.05	0.03	0.03	0.02	0.02	0.02	0.02	0.04

Notes to Table: This table reports the summary statistics of filtered crude oil options contracts for forecast horizons of 1, 2, 3, 4, 5, and 6 weeks; 2, 3, 4, 5, 6, and 9 months; and 1 year. A target observation date is identified by subtracting the appropriate number of days (weekly horizons) or months (monthly and 1-year horizons) from the expiration date, according to Bliss and Panigirtzoglou (2004). If there are no options traded on the target observation date, the nearest options trading date is determined. If this nearest trading date differs from the target observation date by no more than 3 days for weekly horizons or 4 days for monthly and 1-year horizons, that date is substituted for the original target date. If no sufficiently close trading date exists, that expiry is excluded from the sample for that horizon.

Table 2: Estimates of Market Risk Aversion

Forecast Horizon	**1 week**	**2 weeks**	**3 weeks**	**4 weeks**	**5 weeks**	**6 weeks**
n	181	213	221	227	226	227
Panel A. Risk Neutral Density						
p-value	0.2497	0.0407	0.4214	0.3093	0.0610	1.17E-06
μ	0.0932	0.0948	0.1407	0.1283	0.1127	0.1104
ρ	0.1018	0.1403	0.0223	0.0548	0.1113	0.2964
σ	1.0222	0.9922	1.0215	0.9747	0.9952	0.9933
Panel B. Physical Density Calculated Using Power Utility						
p-value	0.4967	0.0712	0.9679	0.8239	0.1337	4.55E-06
μ	0.0058	0.0014	0.0011	0.0028	0.0066	0.0013
ρ	0.1054	0.1796	0.0188	0.0606	0.1545	0.3339
σ	1.0262	1.0041	1.0200	1.0111	1.0086	0.9951
MRRA (Γ)	2.0262	1.1807	1.3856	1.1877	0.8759	1.0009
Panel C. Physical Density Calculated Using Exponential Utility						
p-value	0.4005	0.0615	0.9363	0.7215	0.1107	2.81E-06
μ	0.0266	0.0149	0.0077	0.0168	0.0220	0.0238
ρ	0.1124	0.1823	0.0241	0.0699	0.1579	0.3372
σ	1.0303	1.0054	1.0249	1.0161	1.0103	0.9962
MARA (Γ')	0.0409	0.0255	0.0367	0.0291	0.0196	0.0195
MRRA	1.5360	0.8839	1.2604	0.9879	0.6670	0.6620

Notes to Table: This table presents the results of the modified Berkowitz test of the risk neutral and subjective probability density functions to forecast the futures distribution of the prices of the underlying asset. Probability density functions are constructed by adjusting the risk neutral probability density functions using the appropriate utility function. Risk aversion parameters in the utility function were selected to maximize the Berkowitz likelihood ratio (LR) statistic. I report the p-value of the LR test for i.i.d normality of the inverse-normal transformed inverse-probability transforms of the realizations.

Table 3: Bootstrap Estimation Results

Forecast Horizon		**1 week**	**2 weeks**	**3 weeks**	**4 weeks**	**5 weeks**	**6 weeks**
Panel A. Physical Density Calculated Using Power Utility							
MRRA		2.0262***	1.1807***	1.3856***	1.1877***	0.8759***	1.0009***
Bootstrap	Minimum	-2.93	-2.12	-2.38	-1.18	-1.49	-1.06
	Mean	1.58	1.23	1.09	1.20	0.84	0.86
	Median	1.66	1.24	1.13	1.21	0.84	0.86
	Maximum	7.19	4.46	3.94	3.75	3.39	3.08
	Standard deviation	1.58	1.10	0.91	0.77	0.70	0.66
Panel B. Physical Density Calculated Using Exponential Utility							
MRRA		1.5360***	0.8839***	1.2604***	0.9879***	0.6670***	0.6620***
Bootstrap	Minimum	-3.66	-1.88	-1.65	-1.39	-1.40	-1.06
	Mean	1.65	1.17	1.03	1.14	0.80	0.82
	Median	1.63	1.18	1.05	1.16	0.82	0.83
	Maximum	5.99	3.96	3.36	3.35	3.11	2.82
	Standard deviation	1.53	1.01	0.81	0.71	0.65	0.58

Notes to Table: This table reports the statistics of market risk aversion coefficients (MRRA) from bootstrap tests using futures options maturing in four weeks. ***, **, * represent significance levels of 1%, 5%, and 10%, respectively.

Table 4: Subsample Estimation Results

Subsamples	Mean Volatility	Mean Speculation	MRRA	RN LR p-value	Physical LR p-value
Low Speculation Period	0.3048	1.0261	1.6045***	0.1356	0.3297
High Speculation Period	0.3579	1.082	1.0506***	0.6438	0.8733
Jan. 1990 – Dec. 2002	0.3251	1.0355	1.2940***	0.3149	0.7377
Jan. 2003 – Dec. 2008	0.3495	1.1048	0.8503***	0.6771	0.7705
Jan. 1990 – Dec. 2004	0.3287	1.0391	1.4884***	0.2555	0.8568
Jan. 2005 – Dec. 2008	0.3419	1.1085	-0.0644**	0.6355	0.6359

Notes to Table: This table reports subsample estimation results of market relative risk aversion (MRRA) coefficients for forecast horizon of four weeks. physical probability density functions are implied using power utility function. The mean volatility level and mean speculation index level for each subsample period are also reported. ***, **, * represent significance levels of 1%, 5%, and 10%, respectively.

Table 5: Estimation of State-Dependent Market Risk Aversion

a	b	p-value	μ	ρ	σ
Panel A. Risk Neutral Density					
		0.3093	0.1050	0.0564	1.0095
Panel B. Physical Density Calculated by Power Utility Function					
1.2603***	-24.4638*	0.8728	0.0022	0.0545	1.0066
Panel C. Physical Density Calculated by Exponential Utility Function					
0.0610***	-1.0543***	0.9683	-0.0016	0.0330	0.9955

Notes to Table: This table reports the estimation results for the coefficients in the state-dependent market risk aversion specification and the corresponding p-values. a and b are parameters defined in equation (3.16) and equation (3.17). Results are for a forecast horizon of four weeks. ***, **, * represent significance levels of 1%, 5%, and 10%, respectively.

Table 6: Predictive Regressions

Predictor	**c**	**d**	$\mathbf{R^2}$
Lagged Futures Return	0.0798	0.1257	0.0152
	(0.0755)	(0.0678)*	
RP Using Power Utility	0.132	-11.5722	0.0005
	(0.1406)	(35.6123)	
RP Using Exp. Utility	0.1316	-0.0149	0.0002
	(0.1949)	(0.0701)	
State-dependent RP Using Power Utility	-0.0522	1.0845	0.0208
	(0.1004)	(0.4978)**	
State-dependent RP Using Exp. Utility	-0.0884	1.3376	0.068
	(0.0859)	(0.3316)***	
Speculation Index	4.4183	-4.1079	0.0163
	(2.2506)**	(2.1365)*	
Lagged ATM Volatility	0.1246	-0.0944	0.0001
	(0.2381)	(0.6845)	

Notes to Table: This table reports the coefficients c and d in predictive regression equation (5.3) and the R-square of the regression. Results are for forecast horizon of four weeks. ***, **, * represent significance levels of 1%, 5%, 10%.

Table 7. Robustness Check

	MRRA
Panel A. Whole Sample Estimation	
Jan. 1990 - Dec. 2008	1.8026
Panel B. Subsample Estimation	
Jan. 1990 - Dec. 2002	2.0832
Jan. 2003 - Dec. 2008	1.2085
Jan. 1990 - Dec. 2004	2.0685
Jan. 2005 - Dec. 2008	0.8146
Panel C. State-Dependent Estimation	
a	0.5334
b	-4.4784

Notes to Table: This table reports the estimation results for the market relative risk aversion coefficients following the methodology in Liu, Shackleton, Taylor, and Xu (2005). The physical density for above estimates is calculated assuming CARA power utility function. a and b are parameters defined in equation (3.16). The results are for a forecast horizon of four weeks.