Short Sale Constraints, Correlation and Market Efficiency

Abstract

This paper models a market where short sales are prohibited and investors have heterogeneous beliefs on asset values. We show that short sale constraints may cause overpricing, the magnitude of which depends on not only investors' opinion dispersion on the value of the particular asset, but also on the correlation of the particular asset to other assets and the investors' opinion dispersion for the values of those other assets.

JEL Classification Codes: G10, G14

Keywords: short sale constraints, opinion dispersion, correlation, market efficiency

1. Introduction

Short sale constraints are often considered responsible for allowing optimistic investors to drive prices up, as pessimists sit in the sidelines; the resulted overpricing is expected to increase with high levels of opinion dispersion. This argument, originally hypothesized by Miller (1977), has been widely supported by empirically. While the theoretical literature has occasionally challenged the overpricing argument, there has been very little research on determining other potential drivers (beyond opinion dispersion), which may be contributing to the empirically documented overpricing. Our paper explores the role of correlation in an effort to close this gap in the literature.

The theoretical literature on the effects of short sale constraints on market efficiency starts with Miller's (1977) seminal work, which shows that overpricing is inevitable in market with short sale constraints and that this overpricing depends on opinion dispersion. Harrison and Kreps (1978) present a model of a short sale constrained market with heterogeneous investors often willing to pay a price even higher than the most optimistic investor's view. Figlewski(1981) also shows short sale constrained assets with adverse information to be heavily overpriced and argues that price inefficiencies will arise, even if investors correct their expectations at the market level. Chen, Hong and Stein (2002) extend Miller's (1977) work modeling opinions as uniformly distributed; they show that opinion dispersion leads to overpricing if short sale constraints bind at least some investors and if opinion dispersion is above a minimum threshold.

Nevertheless, the existing theoretical literature offers some possible reasons against overpricing. Diamond and Verrecchia (1987) assume rational expectations and predict that although short sale constraints reduce the speed of price adjustment to private information, price discovery can eventually be achieved. Also, Jarrow (1980) extends Miller's (1977) model to include multiple assets and heterogeneous expectations over both the expected values and the covariance matrix. He shows that the pricing effect of short sale constraints can be complex, with assets being either overpriced or underpriced. However, assets will never be underpriced when investors have different beliefs about the mean future values but agree on the variance covariance matrix of asset returns.

This paper follows Chen, Hong and Stein's (2002) methodology, but includes two risky assets adopting Jarrow's (1980) special case, where investors disagree on expected asset values, but agree on the covariance matrix. Jarrow (1980) shows that the effectiveness of the price discovery process is not conclusive: assets can be either fairly priced or overpriced. Exploring the drivers of this potential overpricing, we find each asset's overpricing to depend not only on the investors' opinion dispersion for the value of the particular asset, but to also be positively associated with its correlation to other assets, as well as the investors' opinion dispersion for the values of those other assets.

The rest of the paper is organized as follows: Section 2 introduces the model and solves each investor's utility maximization problem when no short no short sale constraints are present. Section 3 presents the results, while section 4 discusses policy and empirical implications. Section 5 concludes the paper.

2. The Model

The model describes a market with a risk free asset, two risky assets and n short sale constrained investors with CARA utility function and heterogeneous beliefs. Returns of risky assets are multivariate normally distributed. At $t= 0$, investors form expectations F_{j} for the value of each risky asset j, j=1,2 at the terminal period $t= 1$.Investors disagree on the expected terminal value of the first asset F_1, with their opinions uniformly distributed between $(F_1{-}H_1, F_1{+}H_1)$, but agree on F_2. Therefore, in the absence of short sale constraints, investors should be on average correct in their assessment and both assets would be priced correctly. The supply of the risky assets is fixed and therefore, asset prices at $t= 0$ are driven by investor demand. The cumulative risk free rate is given by R_f=1. In more detail, every investor i maximizes the utility of his terminal wealth $\tilde{W}_i$:

$$Max\{E(-e^{-b\tilde{W}_i})\}\ s.t.\tilde{W}_i{=}W_{0i}\tilde{R}_{pi}, \tag{1}$$

$$\tilde{R}_{pi}{=}R_f{+}w_{1i}(\tilde{R}_{1i}{-}R_f){+}w_{2i}(\tilde{R}_{2i}{-}R_f) \tag{2}$$

$$\tilde{R}_{1i}{=}\frac{\left(\tilde{V}_{1i}{-}P_1\right)}{P_1}+1, \quad \tilde{V}_{1i}{=}F_{1i}{+}\widetilde{\varepsilon}_{1i}, \qquad \widetilde{\varepsilon}_{1i}{\sim}N\left(0,1\right) \tag{3}$$

$$\tilde{R}_{2i}=\frac{\left(\tilde{V}_{2i}-P_2\right)}{P_2}+1,\quad \tilde{V}_{2i}=F_{2i}+\rho\widetilde{\varepsilon}_{1i}+\sqrt{1-\rho^2}\widetilde{\varepsilon}_{2i}, \qquad \widetilde{\varepsilon}_{2i}\sim N(0,1) \tag{4}$$

$$F_{1i}\sim U(F_1-H_1,F_1+H_1),\ \text{and}\ F_{2i}=F_2, \tag{5}$$

with V_{ji}= terminal value of asset j, $\tilde{R}_{\text{ji}}$= j's cumulative return, w_{ji}= j's weight, H_1= opinion dispersion, W_{0i}= initial wealth, b= absolute risk aversion. In the absence of short sale constraints, the individual demand functions are:

$$Q_{1i}=\frac{P_1-P_2\rho-F_{1i}+\rho F_2}{b(\rho^2-1)},\ \text{and}\ Q_{2i}=\frac{P_2-P_1\rho-F_2+\rho F_{1i}}{b(\rho^2-1)}. \tag{6}$$

Individual investing behavior is affected by the level of correlation between the two risky assets. We describe investor i's trading behavior when $\rho>0$, $\rho=0$, $\rho<0$, where the aggregate demand function for every asset is constrained to be positive.

Case 1: $\rho>0$

If $F_{1i}<P_1+(F_2-P_2)\ \rho$, investor i will short asset 1 and buy asset 2.

If $P_1+(F_2-P_2)\rho<F_{1i}<\frac{(F_2-P_2)}{\rho}+P_1$, investor i will buy both assets.

If $F_{1i}>\frac{(F_2-P_2)}{\rho}+P_1$, investor i will buy asset 1 and short asset 2.

Case 2: $\rho=0$

If $P_2 < F_2$ and $P_1 < F_{1i}$, investor i will buy both assets.

If $P_2 < F_2$ and $P_1 > F_{1i}$, investor i will short asset 1 and buy asset 2.

Case 3: $\rho < 0$

If $F_{1i} > P_1 + (F_2 - P_2)\,\rho$, investor i will buy both assets.

If $\frac{(F_2 - P_2 + P_1 \rho}{\rho} < F_{1i} < P_1 + F_2 \rho - P_2$, investor i will short asset 1 and buy asset 2.

If $F_{1\mathrm{i}} < \frac{F_2 - P_2 + P_1 \rho}{\rho}$, investor i will short both assets.

3. Results

The aggregate unconstrained demand functions are:

$$Q_1{}^u = \frac{1}{2H_1} \int_{F_1 - H_1}^{F_1 + H_1} \frac{P_1 - P_2 \rho - F_{1i} + \rho F_2}{b(\rho^2 - 1)} dF_{1i} = \frac{P_1 - P_2 \rho - F_1 + \rho F_2}{b(\rho^2 - 1)}, \tag{7}$$

$$Q_2{}^u = \frac{1}{2H_1} \int_{F_1 - H_1}^{F_1 + H_1} \frac{P_2 - P_1 \rho - F_2 + \rho F_{1i}}{b(\rho^2 - 1)} dF_{1i} = \frac{P_2 - P_1 \rho - F_2 + \rho F_1}{b(\rho^2 - 1)}. \tag{8}$$

The integration limits of the demand function under a short sale ban depend on the level of opinion dispersion and correlation. The appendix describes the aggregate constrained demand functions for all cases ($\rho > 0$, $\rho = 0$, $\rho < 0$) and their subcases. For example, when $\rho > 0$ and opinion dispersion is low enough (case 1.1) so that $P_1 + (F_2 - P_2)\ \rho < F_1 - H_1 < F_1 + H_1 < \frac{(F_2 - P_2)}{\rho} + P_1$, investors do not desire to short sell either asset and therefore a short sale ban would not affect investor behavior or prices. However, when for example risky assets are positively correlated and opinion dispersion is high enough (case 1.2) so that $F_1{-}H_1 < P_1{+}(F_2{-}P_2)\rho < \frac{(F_2-P_2)}{\rho}{+}P_1{<}F_1{+}H_1$, investors who value asset 1 in the interval $(F_1{-}H_1, P_1{+}(F_2{-}P_2)\rho)$ would short sell it, were they not restricted; thus, they now just demand ${Q_2}^{c1} = \frac{(F_2-P_2)}{b}1$ units of asset 2. Similarly, in the absence of a short sale ban, investors who value asset 1 in the interval $(\frac{(F_2-P_2)}{\rho}{+}P_1, F_1{+}H_1)$ want to short asset 2; under the ban, they just hold a long position in asset 1. Investors with asset 1 valuation in the interval $(P_1{+}\,(F_2{-}P_2)\,\rho, \frac{(F_2-P_2)}{\rho}{+}P_1)$ hold only long positions.

Noticeably, higher non-zero correlation shrinks the integration boundaries for $Q_1, Q_2{>}0$. Since investors view prices as fixed, higher correlation increases the probability that investors would desire short positions and higher opinion dispersion (H_1) translates in more investors willing to short; thus, the short sale constraints' effective binding strength increases. Overpricing is[2]:

$$ {P_1}^c{-}{P_1}^u{=}\frac{b({Q_1}^c{-}{Q_1}^u{+}{Q_2}^c\rho{-}{Q_2}^u\rho)}{n}, \tag{9} $$

$$ {P_2}^c{-}{P_2}^u{=}\frac{b({Q_2}^c{-}{Q_2}^u{+}{Q_1}^c\rho{-}{Q_1}^u\rho)}{n}. \tag{10} $$

where ${Q_j}^c, {Q_j}^u$ are the constrained and unconstrained aggregate demand functions and ${P_j}^c, {P_j}^u$ the corresponding equilibrium prices for asset j, j=1,2.

[1] Pennacchi, G., 2006. Theory of Asset Pricing, Pearson Addison Wesley, page 50.
[2] The proof can be provided by the authors upon request.

Table 1

	(1)	(2)	(3)	(4)	(5)	(6)
	$P_1^c - P_1^u$	$P_2^c - P_2^u$	$\frac{\partial(P_1{}^c - P_1{}^u)}{\partial H_1}$	$\frac{\partial(P_2{}^c - P_2{}^u)}{\partial H_1}$	$\frac{\partial(P_1{}^c - P_1{}^u)}{\partial \rho}$	$\frac{\partial(P_2{}^c - P_2{}^u)}{\partial \rho}$
			$\rho > 0$			
1.1	0	0	0	0	0	0
1.2	+	+	+	+	+	+
1.3	+	0	+	0	+	0
1.4	0	+	0	+	0	+
			$\rho = 0$			
2.1	0	0	0	0	0	0
2.2	+	0	+	0	+	0
			$\rho < 0$			
3.1	0	0	0	0	0	0
3.2	+	0	+	0	+	0
3.3	+	+	+	+/-	-	+

Table 1 reports the sign of each asset's mispricing for all subcases in the appendix. The first section of the table presents subcases for positive correlation, the second for zero and the third for negative correlation. The first two columns show that mispricing for assets 1 and 2 respectively is always positive or zero. The effect of asset 1's opinion dispersion and the correlation effect on both assets' overpricing are measured with the corresponding derivatives, whose signs are reported in columns (3)-(6). Columns (3), (4) show opinion dispersion to have a positive effect on the risky assets' overpricing almost always. Similarly, columns (5) and (6) show that correlation typically increases overpricing for both assets. Results are summarized in the following two propositions:

Proposition 1: When short sale constraints bind, opinion dispersion on an asset's valuation leads to overpricing for this asset and any other asset, correlated to it. This result is partly in agreement with Miller(1977) and Chen et al. (2002), who argue that some minimum level of opinion dispersion is required to cause overpricing. Our model shows that an asset can be overpriced even if everyone agrees on its value, as long as short sale constraints bind and investors disagree on the value of another asset with correlated returns to the original asset. Moreover, overpricing will be positively affected by the level of this disagreement. The intuition behind this finding is that high opinion dispersion for the value of the first asset translates in an increased number of investors who would be willing to short the second asset because of diversification benefits arising from the correlation of the two assets. Short sale constraints restrict this behavior causing overpricing. Opinion dispersion on an asset's value will have no effect on the value of any other asset if the two assets are uncorrelated.

Proposition 2: In a market with short sale constraints, the correlation between risky assets has a positive effect on overpricing for all assets, as long as investors disagree on the value of at least one asset. The rationale behind this lies in that high correlation translates in lower diversification benefits, which alters investors' desired allocation: at low correlation levels, investors will have a stronger incentive to long both assets as the diversification benefit is high. As correlation increases, the diversification benefit declines; then, pessimist investors are more likely to want to short the asset with lower valuation. At very high correlation levels, assets are viewed as substitutes and an increasing number of investors would be willing to short one or both assets. Short sale constraints restrict these investors from taking short positions causing overpricing. The case of very negative correlation and high opinion dispersion is an exception: then, very pessimist investors would want to short both assets and therefore an increase in correlation would reduce overpricing. This is unlikely to be observed in practice, as highly negatively correlated equity returns are uncommon.

4. Policy and empirical implications

Our results have important policy implications relating to the application of short sale constraints. They show that a high correlation between asset returns in the markets with short sale constraints translates in higher magnitude of overpricing. This is very important, as short sale bans are often applied at times of high uncertainty, when market correlation spikes. Our results suggest in these times the adverse effects of short sale constraints on market efficiency will be more pronounced; the effort to avoid market panic results in higher market inefficiencies than previously thought. Our findings also suggest that short sale constraints are expected to result in different levels of market inefficiencies, depending on the characteristics of the underlying market. Therefore, international equity markets with high concentration should suffer more from market inefficiencies is short sales are constrained. Finally, our findings suggest that the interdependence of equity assets provides the rationale for the study of the pricing effects of short sale constraints at the market level in addition to just the short sale constrained assets. The empirical confirmation of our theoretical results remains and is part of our future research plans.

5. Conclusion

This paper models a market with heterogeneous investors and two risky assets. Consistent to previous literature, short sale constraints cause asset overpricing, which increases as opinion dispersion becomes more pronounced. Opinion dispersion over the value of one asset may, also, cause overpricing to another asset, provided the two assets' returns are correlated. Finally, correlation between asset returns will generally have a positive effect on overpricing for all assets. This is a novel finding with potentially crucial policy implications, especially for international equity markets, which often have high concentration of their equity in a few industries. There, according to our model, overpricing will be more pronounced if short sale constraints are applied.

References

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Appendix

1. Positively correlated assets: $\boldsymbol{\rho}$**>0**

1.1. $\boldsymbol{\rho}>0$ and $\mathbf{P_1+(F_2-P_2)\boldsymbol{\rho} < F_1-H_1<F_1+H_1<(F_2-P_2)\,\boldsymbol{\rho}+P_1}$

$$Q_1{}^c{=}\frac{1}{2H_1}\int_{F_1-H_1}^{F_1+H}Q_{1i}dF_{1i}=Q_1{}^u, \tag{A.1}$$

$$Q_2{}^c{=}\frac{1}{2H_1}\int_{F_1-H_1}^{F_1+H}Q_{2i}dF_{1i}Q_2{}^u. \tag{A.2}$$

1.2. $\boldsymbol{\rho}>0$ and $\mathbf{F_1-H_1<P_1+(F_2-P_2)\,\boldsymbol{\rho}<\frac{(F_2-P_2)}{\boldsymbol{\rho}}+P_1<F_1+H_1}$

$$Q_1{}^c{=}\frac{1}{2H_1}\int_{P_1+(F_2-P_2)\rho}^{P_1+\frac{(F_2-P_2)}{\rho}}Q_{1i}dF_{1i}{+}\frac{1}{2H_1}\int_{P_1+\frac{(F_2-P_2)}{\rho}}^{F_1+H_1}\frac{(F_1-P_1)}{b}dF_{1i}{>}Q_1{}^u, \tag{A.3}$$

$$Q_2{}^c{=}\frac{1}{2H_1}\int_{F_1-H_1}^{P_1+(F_2-P_2)\rho}\frac{(F_2-P_2)}{b}dF_{1i}{+}\frac{1}{2H_1}\int_{P_1+(F_2-P_2)\rho}^{P_1+\frac{(F_2-P_2)}{\rho}}Q_{2i}dF_{1i}{>}Q_2{}^u. \tag{A.4}$$

1.3. $\boldsymbol{\rho}>0$ and $\mathbf{F_1-H_1<P_1+(F_2-P_2)\,\boldsymbol{\rho}<F_1+H_1<\frac{F_2-P_2}{\boldsymbol{\rho}}+P_1}$

$$Q_1{}^c{=}\frac{1}{2H_1}\int_{P_1+(F_2-H_2)\rho}^{F_1+H_1}Q_{1i}dF_{1i}{>}Q_1{}^u, \tag{A.5}$$

$$Q_2{}^c = \frac{1}{2H_1}\int_{F_1-H_1}^{P_1+(F_2-P_2)\rho}\frac{(F_2-P_2)}{b}dF_{1i} + \frac{1}{2H_1}\int_{P_1+(F_2-P_2)\rho}^{F_1+H_1} Q_{2i}dF_{1i} > Q_2{}^u. \tag{A.6}$$

1.4. $\boldsymbol{\rho}>0$ and $\mathbf{P_1}+(\mathbf{F_2}-\mathbf{P_2})\boldsymbol{\rho}<\mathbf{F_1}-\mathbf{H_1}<\frac{(\mathbf{F_2}-\mathbf{P_2})}{\boldsymbol{\rho}}+\mathbf{P_1}<\mathbf{F_1}+\mathbf{H_1}$

$$Q_1{}^c = \frac{1}{2H_1}\int_{F_1-H_1}^{\frac{(F_2-P_2)}{\rho}+P_1} Q_{1i}dF_{1i} + \frac{1}{2H_1}\int_{\frac{(F_2-P_2)}{\rho}+P_1}^{F_1+H_1}\frac{(F_1-P_1)}{b}dF_{1i} > Q_1{}^u, \tag{A.7}$$

$$Q_2{}^c = \frac{1}{2H_1}\int_{F_1-H_1}^{\frac{(F_2-P_2)}{\rho}+P_1} Q_{2i}dF_{1i} > Q_2{}^u. \tag{A.8}$$

2. Uncorrelated assets: $\boldsymbol{\rho} = \mathbf{0}$

2.1. $\boldsymbol{\rho}=\mathbf{0}$ and **P2<F2** and $\mathbf{P_1}<\mathbf{F_1}-\mathbf{H_1}<\mathbf{F_1}+\mathbf{H_1}$

$$Q_1{}^c = \frac{1}{2H_1}\int_{F_1-H_1}^{F_1+H_1}\frac{(F_{1i}-P_1)}{b}dF_{1i} = Q_1{}^u, \tag{A.9}$$

$$Q_2{}^c = \frac{1}{2H_1}\int_{F_1-H_1}^{F_1+H_1}\frac{(F_2-P_2)}{b}dF_{2i} = Q_2{}^u. \tag{A.10}$$

2.2. $\boldsymbol{\rho}=\mathbf{0}$ and **P2<F2** and $\mathbf{F_1}-\mathbf{H_1}<\mathbf{P_1}<\mathbf{F_1}+\mathbf{H_1}$

$$Q_1{}^c = \frac{1}{2H_1}\int_{P_1}^{F_1+H_1} \frac{(F_{1i}-P_1)}{b} dF_{1i} > Q_1{}^u, \tag{A.11}$$

$$Q_2{}^c = \frac{1}{2H_1}\int_{F_1-P_1}^{F_1+H_1} \frac{(F_2-P_2)}{b} dF_{2i} = Q_2{}^u. \tag{A.12}$$

3. Negatively correlated assets: $\boldsymbol{\rho} < \mathbf{0}$

3.1. $\boldsymbol{\rho}<0$ and $\mathbf{P_1+F_2\boldsymbol{\rho}-\ P_2\ \boldsymbol{\rho}<P_1+\frac{(F_2-P_2)}{\boldsymbol{\rho}}<F_1-H_1<F_1+H_1}$

$$Q_1{}^c = \frac{1}{2H_1}\int_{F_1-H_1}^{F_1+H_1} Q_{1i} dF_{1i} = Q_1{}^u, \tag{A.13}$$

$$Q_2{}^c = \frac{1}{2H_1}\int_{F_1-H_1}^{F_1+H_1} Q_{1i} dF_{1i} = Q_2{}^u. \tag{A.14}$$

3.2. $\boldsymbol{\rho}<0$ and $\mathbf{\frac{(F_2-P_2+P_1\boldsymbol{\rho})}{\boldsymbol{\rho}}<F_1-H_1<P_1+F_2\boldsymbol{\rho}-P_2\boldsymbol{\rho}<F_1+H_1}$

$$Q_1{}^c = \frac{1}{2H_1}\int_{P_1+(F_2-P_2)\rho}^{F_1+H_1} Q_{1i} dF_{1i} > Q_1{}^u, \tag{A.15}$$

$$Q_2{}^c = \frac{1}{2H_1}\int_{F_1-H_1}^{P_1+(F_2-P_2)\rho} \frac{(F_2-P_2)}{b}\ dF_{1i} + \frac{1}{2H_1}\int_{P_1+(F_2-P_2)\rho}^{F_1+H_1} Q_{2i} dF_{1i} > Q_2{}^u. \tag{A.16}$$

3.3. $\boldsymbol{\rho}<0$ and $\mathbf{F_1}-\mathbf{H_1}<\frac{(\mathbf{F_2}-\mathbf{P_2})}{\boldsymbol{\rho}}+\mathbf{P_1}<\mathbf{P_1}+(\mathbf{F_2}-\mathbf{P_2})\,\boldsymbol{\rho}<\mathbf{F_1}+\mathbf{P_1}$

$$Q_1{}^c=\frac{1}{2H_1}\int_{P_1+(F_2-P_2)\rho}^{F_1+H_1} Q_{1i}dF_{1i}>Q_1{}^u, \tag{A.17}$$

$$Q_2{}^u=\frac{1}{2H_1}\int_{\frac{(F_2-P_2)}{\rho}+P_1}^{P_1+(F_2-P_2)\rho} \frac{(F_2-P_2)}{b}dF_{1i}+\frac{1}{2H_1}\int_{P_1+(F_2-P_2)\rho}^{F_1+H_1} Q_{2i}dF_{1i}>Q_2{}^u. \tag{A.18}$$